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for
ELF/VLF Electromagnetic Detection and Characterization of Deeply Buried Targets

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This is the Final Report for the SBIR Phase II effort entitled ?ELF/VLF Electromagnetic Detection and Characterization of Deeply Buried Targets? sponsored by the Defense Advanced Research Projects Agency. The various electromagnetic calculational tools coded as MATLAB routines delivered with the report constitute the goal of our effort. The report is in five parts: (1) a description of the Sommerfeld routines; (2) a description of the menu interface routines; (3) a description of the Hill/King buried wire routines; (4) an appendix of the Sommerfeld integrals used; (5) an appendix describing the CDROM containing the MATLAB codes.

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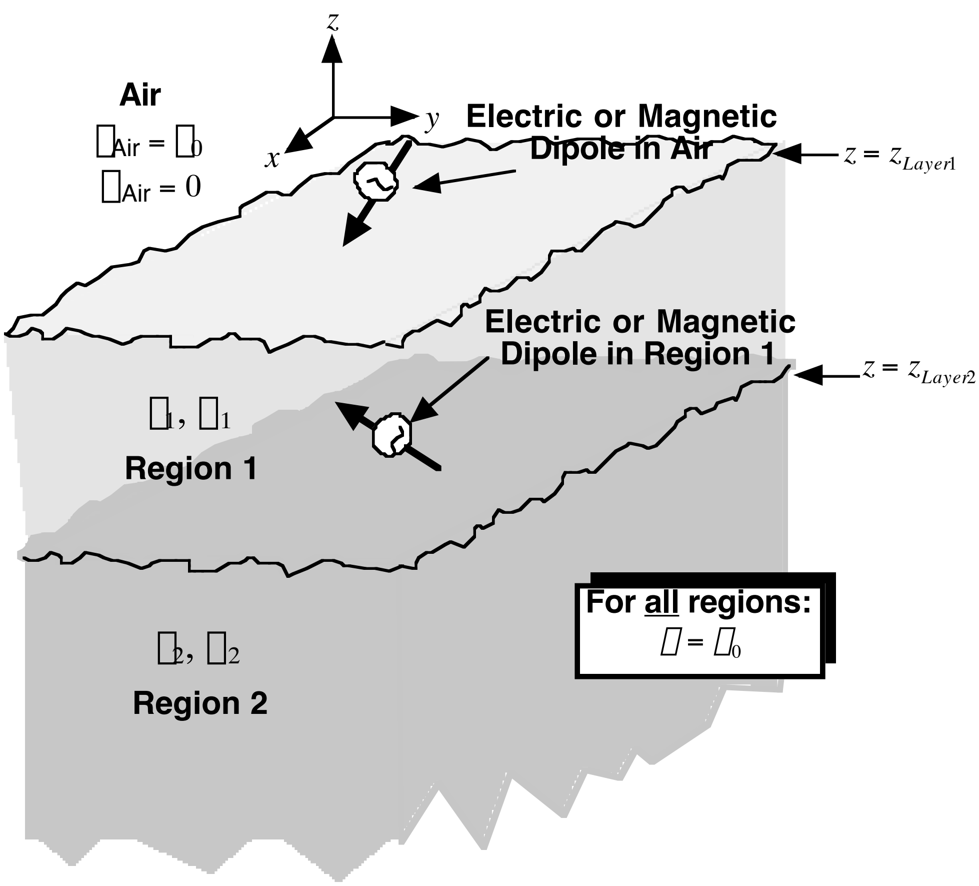
ELF/VLF Electromagnetic Detection and Characterization of Deeply Buried Targets

This is the Final Report for the SBIR Phase II effort entitled "*ELF/VLF Electromagnetic Detection and Characterization of Deeply Buried Targets*" sponsored by the Defense Advanced Research Projects Agency. The various electromagnetic calculational tools coded as MATLAB routines delivered with the report constitute the goal of our effort. The report is in five parts: (1) a description of the Sommerfeld routines; (2) a description of the menu interface routines; (3) a description of the Hill/King buried wire routines; (4) an appendix of the Sommerfeld integrals used; (5) an appendix describing the CD-ROM containing the MATLAB codes.

Sommerfeld Calculation Routines for a Two-Layer Earth

The configuration for the Sommerfeld routines is sketched below:



To calculate the electric or magnetic fields due to a dipole at a set of points, one of two

MATLAB function routines are used: `getElecDipoleField` or `getMagDipoleField`; these functions are called in MATLAB as:

```
[EField,HField] = ...
            getElecDipoleField(EFieldOn,HFieldOn,ILdipMom,...
                  RSourcePoint,REvalPoints,freqVal,...
                  zLayer1,eDielec1,Conduct1,zLayer2,eDielec2,Conduct2);
```

or

```
[EField,HField] = ...
            getMagDipoleField(EFieldOn,HFieldOn,IAdipMom,...
            RSourcePoint,REvalPoints,freqVal,...
            zLayer1,eDielec1,Conduct1,zLayer2,eDielec2,Conduct2);
```

The description for most of the arguments is the same for both routines; the routines' input arguments differ only in the definition of the dipole moments.

Below are the <u>*input argument descriptions*</u>:

EFieldOn - Logical variable: ON (=1) -> Calculate the E Field;
OFF (=0) -> Don't Calculate the E Field

HFieldOn - Logical variable: ON (=1) -> Calculate the H Field;
OFF (=0) -> Don't Calculate the H Field

RSourcePoint - Location of the dipole source in meters for user defined Cartesian coordinates; RSourcePoint(1,3),
i.e. RSourcePoint = [xsrc,ysrc,zsrc]

REvalPoints - Points to evaluate the E and/or H Fields in meters; uses the same (user defined) Cartesian coordinate system as RSourcePoint (above); REvalPoints(1:N,3), where N is the number of eval points
i.e. REvalPoints(1,:) = [xEv(1),yEv(1),zEv(1)],
REvalPoints(2,:) = [xEv(2),yEv(2),zEv(2)],
REvalPoints(3,:) = [xEv(3),yEv(3),zEv(3)],
.... etc.
N.B. For a single call of either routine the REvalPoints selected must all be in a single medium; if the E/H Field values are desired in both Air and Region 1, this may be accomplished in two sequential calls. Also note: Fields cannot be calculated in Region 2, as this is intended to model a substrate below the air and ground regions of sources and observation points

freqVal - Frequency in Hz

zLayer1 - z value [meters] of Air <--> Region 1 interface in the user defined Cartesian coordinate system
eDielec1 - Relative dielectric constant of Region 1 medium
Conduct1 - Conductivity of Region 1 medium [mhos/m]
zLayer2 - z value [meters] of Region 1 <--> Region 2 interface in the user defined Cartesian coordinate system
eDielec2 - Relative dielectric constant of Region 2 medium
Conduct2 - Conductivity of Region 2 medium [mhos/m]
ILdipMom - Electric dipole moment [amp-m]; used only for the function getElecDipoleField
IAdipMom - Magnetic dipole moment [amp-m^2]; used only for the function getMagDipoleField

Below are the _output value descriptions_:

EField - The value of the E Field due to the dipole at the specified evaluation points (REvalPoints) in volt/m. The E Field is given in Cartesian coordinates. So,

EField(1,:) = [Ex,Ey,Ez] at REvalPoints(1,:)
EField(2,:) = [Ex,Ey,Ez] at REvalPoints(2,:)
EField(3,:) = [Ex,Ey,Ez] at REvalPoints(3,:)
.... etc.

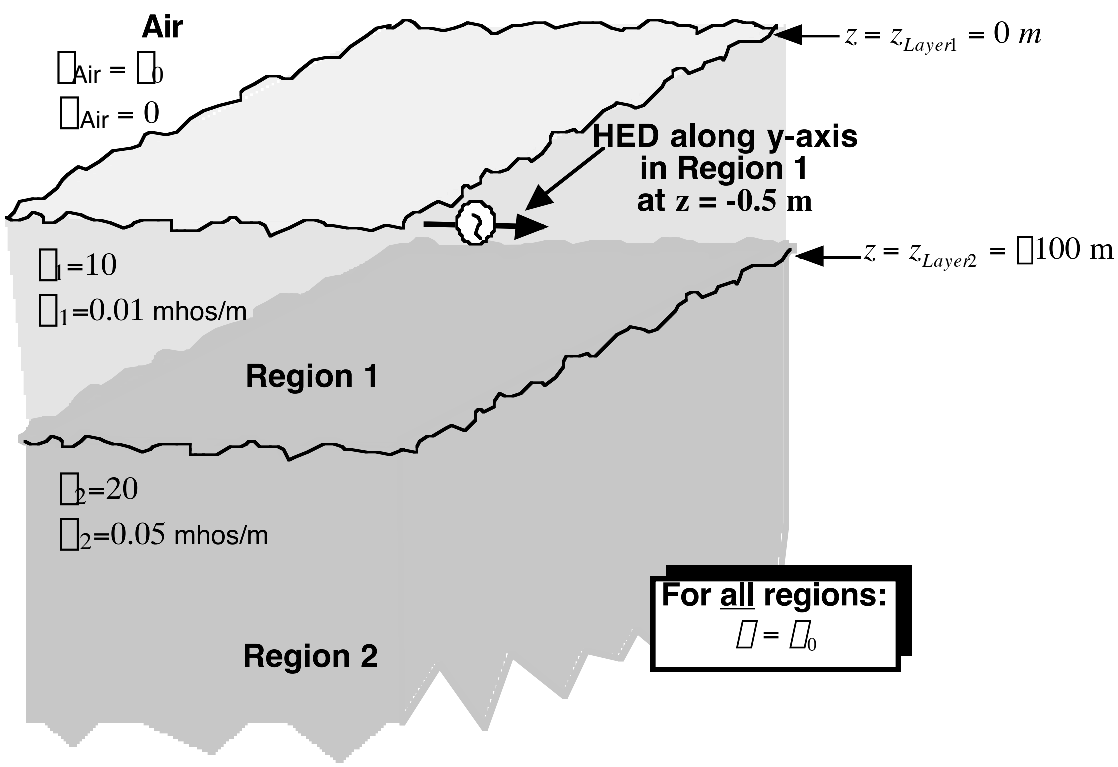
HField - The value of the H Field due to the dipole at the specified evaluation points (REvalPoints) in amp/m. The H Field is given in Cartesian coordinates. The array structure and correspondence between HField and REvalPoints is the same as for EField above.

There are a number of MATLAB functions required by `getElecDipoleField` and `getMagDipoleField`; these functions are supplied on the CD-ROM and they must be kept in the same folder as `getElecDipoleField` and `getMagDipoleField`.

Example: Simple Buried HED Calculation

As a first example we give below the code for calculation of an HED in Region 1 at a depth of 0.5 meter below the surface at 2 kHz. The relative dielectric constant of Region 1 is 10 and the conductivity is 0.01 mhos/m; the relative dielectric constant of Region 2

is 20 and the conductivity is 0.05 mhos/m. The interface between the Air and Region 1 corresponds to z = 0 m; the interface between the Region 1 and Region 2 corresponds to z = -100 m. We assume the electric dipole strength is 0.5 amp-m. The dipole is centered at x = 0 m, y = 0 m, and z = -0.5 m in our coordinate system and the dipole vector is along the y-axis. The electric and magnetic fields are evaluated on a square grid 30 m x 30 m a distance of 1.0 m above the surface at 2 meter intervals. The results are plotted for each field component using the MATLAB `pcolor` command.
The configuration for this example is sketched below.



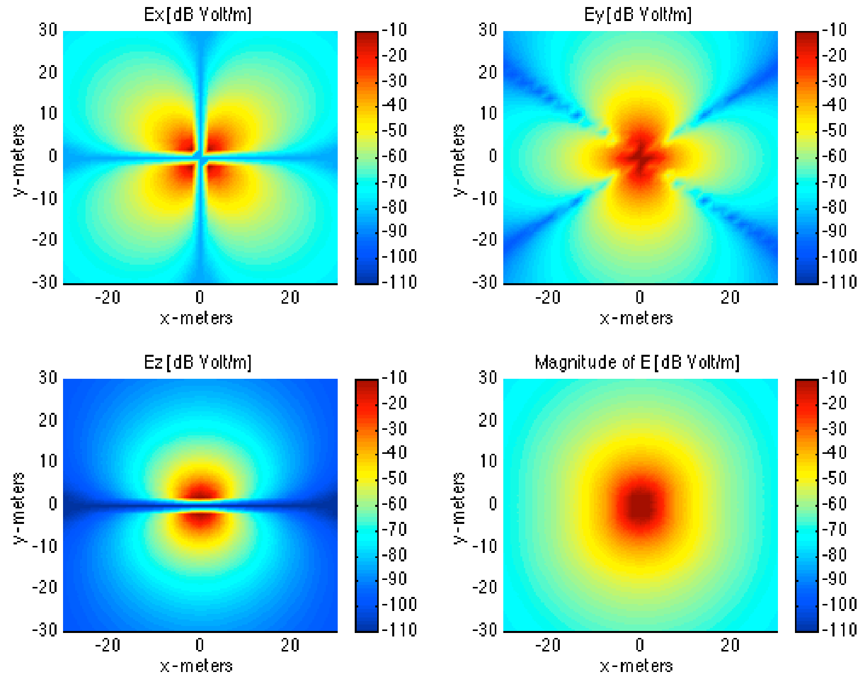
The MATLAB code for this calculation is:

```
%***********************************************************************

%        Set flags to calculate both E Field and H Field
EFieldOn = 1;HFieldOn = 1;

%       Frequency = 2 kHz and set dielectric/conductivity for Region 1 & 2
freqVal  =  2000;
```

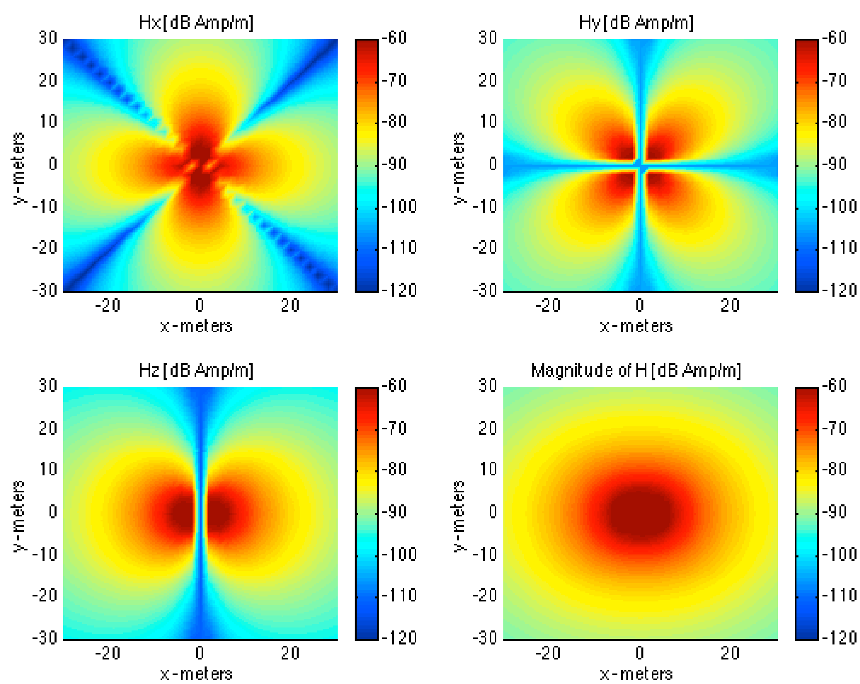
```
eDielec1 = 10;Conduct1 = 0.01;
eDielec2 = 20;Conduct2 = 0.05;

%       Region 1 <--> Air at z = 0 m; Region 2 <--> Region 1 at z = -100 m;
zLayer1 = 0;zLayer2 = -100;

%       Electric dipole moment = 0.5 Amp-m, along y-axis
IL =  0.5;
dipMom = IL.*[0,1,0];

%       Electric dipole moment at x = 0 m, y = 0 m, z = -0.5 m
%       Since the Region 1 <--> Air interface is at z = 0 m (zLayer1 = 0 m),
%       this places the dipole o.5 m below the earth
RSourcePoint = [0,0,-0.5];

%       Set up a square grid of 30 m x 30 m at a distance of 1.0 m above the
%       Region 1 <--> Air interface at 2 meter intervals on which to
%       evaluate the electric and magnetic fields
xx = ((-30):2:30).';yy = ((-30):2:30).';zz = 1.0;
xxTemp = xx*ones(size(yy.'));yyTemp = ones(size(xx))*(yy.');
xxEval = xxTemp(:);yyEval = yyTemp(:);
npts = max(size(xxEval));
REvalPoints = zeros(npts,3);
REvalPoints(:,1) = xxEval;REvalPoints(:,2) = yyEval;
REvalPoints(:,3) = zz.*ones(npts,1);

%       Do calculation to get E/H Fields at the points on the evaluation grid
[EField,HField] = getElecDipoleField(EFieldOn,HFieldOn,dipMom,...
                       RSourcePoint,REvalPoints,freqVal,...
                       zLayer1,eDielec1,Conduct1,zLayer2,eDielec2,Conduct2);
          .
          .
          .
          .
     { MATLAB code to do plots }
          .
          .
          .
          .
*********************************************************************************
```

Note that it is not necessary to just use points calculated in the x-y plane; any set of points is acceptable, as long as all the points in a single call are located in a single layer - either Air or Region 1. We selected the set of points to be in the x-y plane for this example, just to show one useful configuration.

The results of this calculation are shown on the following page in the four color intensity plots for the three E-Field components and the magnitude of the E-Field along with similar plots for the H-Field. We have supplied the complete MATLAB code, `ExampleHED.m`, including the plotting sequences, for this routine on the CD-ROM.



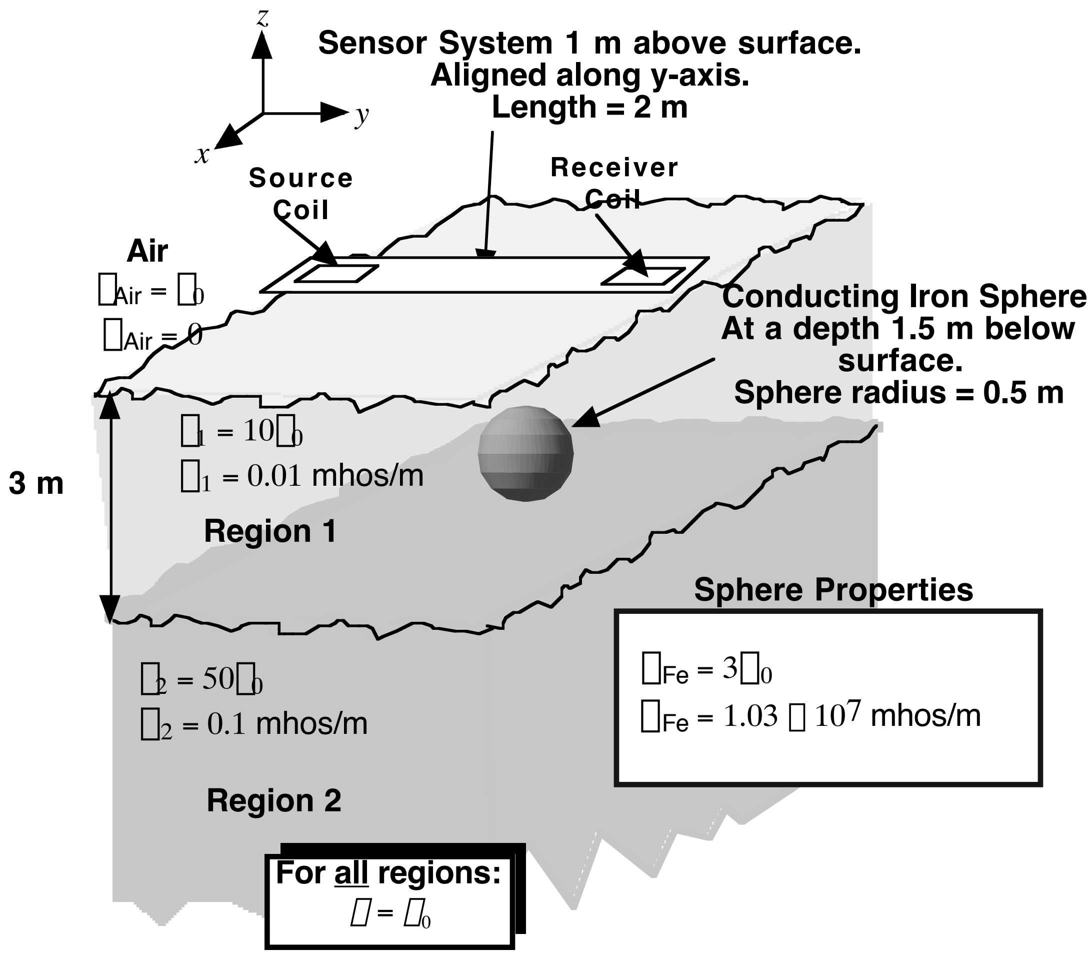
E Field results from HED example - Color intensity scale in dB Volts/m



HField results from HED example - Color intensity scale in dB Amps/m

Example: Generic Geophex GEM Sensor and a Buried Iron Sphere

For the second example we examine a calculation for a sensor system similar to the GEM sensors developed by Geophex, Ltd. of Raleigh, North Carolina (our subcontractor in this effort). The sensor system and measurement configuration are sketched below:



More details on the GEM sensors are given in our Phase 1 Final Report and are, of course, available from Geophex. We are using a somewhat generic version of the GEM in this calculation and the results will therefore be representative in a general sense, but are not meant to be a precise model for any specific version of the GEM sensor.

In this example a 0.5 m radius iron sphere is buried 1.5 meters below the surface and excited by a source coil at one end of the sensor system. This induces a magnetic dipole moment on the sphere, which reradiates and is sensed by the receiver coil at the other end of the sensor system. The receiver also picks up the direct field from the source coil and so the total signal is a sum of the direct field (the field if there sphere were not present) and the perturbed field due to the sphere. Since the GEM is designed to subtract out the direct signal, the effective "measured" signal is the voltage ratio of the perturbed signal to the direct signal; this is usually given in parts-per-million (PPM).

The calculation is done at 1200 Hz; the relative dielectric constant of Region 1 is 10 and the conductivity is 0.01 mhos/m; the relative dielectric constant of Region 2 is 50 and the conductivity is 0.1 mhos/m. The interface between the Air and Region 1 corresponds to z = 0 m; the interface between the Region 1 and Region 2 corresponds to z = -3 m. We assume the magnetic dipole strength of the source coil is 3 amp-m^2. The sphere is centered at x = 0 m, y = 0 m, and z = -1.5 m in our coordinate system. The z component of the perturbed and direct H-Field are evaluated on a square grid 6m x 6m a distance of 1.0 m above the surface at ~0.2 meter intervals. The results are plotted for each field component using the MATLAB `pcolor` command.

The MATLAB routine calls a function **getMagDipMomCond**. This function is supplied on the CD-ROM. The function calculates the induced magnetic dipole moment coefficient for a conducting sphere for a given permeability and dielectric constant. The relation was derived by Wait (*Geophysics, 16, No. 4, 666-672, October 1951* and *Radio Science, Vol. 3 (New Series), No. 10, 1030-1034 October 1968*).

The MATLAB code for this calculation is:

```
%***************************************************************************

SommerfeldGlobals;

%     Source Loop strength 3 amp*m^2 and the loop is held to be
%     parallel to the earth, so the dipole moment is along the z-axis
IA_GEM = 3;
dipMomGEM  =  IA_GEM.*[0,0,1];

%     Source and sensor loops are separated by 2 meters along the y-axis
LGEM = 2;

%     The initial position of the source is 1 meter (z = 1 meter) above
%     the earth; it is separated by LGEM from sensor along y-axis.
RSourcePoint0  =   [0,-LGEM/2,1];
REvalPoints0   =   [0,+LGEM/2,1];

%     Set up measurement grid.
LL = 3;NN = 30;
nx = NN;xMin = -LL;xMax = +LL;dx = 2*LL/(nx-1);
xx = xMin:dx:xMax;
```

```
ny = NN;yMin = -LL;yMax = +LL;dy = 2*LL/(ny-1);
yy = (yMin:dy:yMax).';

freqVal = 1200;
wVal     = 2*pi*freqVal;

eDielec1 = 10;Conduct1 = 0.01;
eDielec2 = 50;Conduct2 = 0.1;

zLayer1 =  0;zLayer2 = -3;

%        Set flags to calculate only the H Field
EFieldOn = 0;HFieldOn = 1;

%     Get Direct Field at Sensor for each of the measurement position points
%     on the grid
REvalPoints = zeros(1,3);
HFieldDirZ = zeros(ny,nx);
for iy=1:ny
      for ix=1:nx
            gridPoint = [xx(1,ix),yy(iy,1),0];
            RSourcePoint = RSourcePoint0 +  gridPoint;
            REvalPoints  = REvalPoints0  +  gridPoint;
            [EFieldDirTemp,HFieldDirTemp] = ...
                  getMagDipoleField(EFieldOn,HFieldOn,dipMomGEM,...
                        RSourcePoint,REvalPoints,freqVal,...
                        zLayer1,eDielec1,Conduct1,zLayer2,eDielec2,Conduct2);
            HFieldDirZ(iy,ix) = HFieldDirTemp(1,3);
      end
end

%     Get Field at Underground Sphere for each of the measurement position
%     points on the grid. Sphere located at x=y=0, z=-1.5 m
REvaUG = [0,0,-1.5];
HFieldUGX = zeros(ny,nx);
HFieldUGY = HFieldUGX;HFieldUGZ = HFieldUGX;
for iy=1:ny
      for ix=1:nx
            gridPoint = [xx(1,ix),yy(iy,1),0];
            RSourcePoint = RSourcePoint0 +  gridPoint;
            [EFieldUGTemp,HFieldUGTemp] = ...
                  getMagDipoleField(EFieldOn,HFieldOn,dipMomGEM,...
                        RSourcePoint,REvaUG,freqVal,...
                        zLayer1,eDielec1,Conduct1,zLayer2,eDielec2,Conduct2);
            HFieldUGX(iy,ix) = HFieldUGTemp(1,1);
            HFieldUGY(iy,ix) = HFieldUGTemp(1,2);
            HFieldUGZ(iy,ix) = HFieldUGTemp(1,3);
      end
end

%     Get Perturbed Field at Sensor for each of the measurement position
%     points on the grid
epsFreeSpace = 8.85E-12;muFreeSpace = 4E-7*pi;
epsOut = epsFreeSpace*(eDielec1+i*Conduct1/(wVal*epsFreeSpace));
muOut  = muFreeSpace;
sigIron = 1.03E7;muIron = 3*muFreeSpace;
epsSph = epsFreeSpace*(0.0+i*sigIron/(wVal*epsFreeSpace));
```

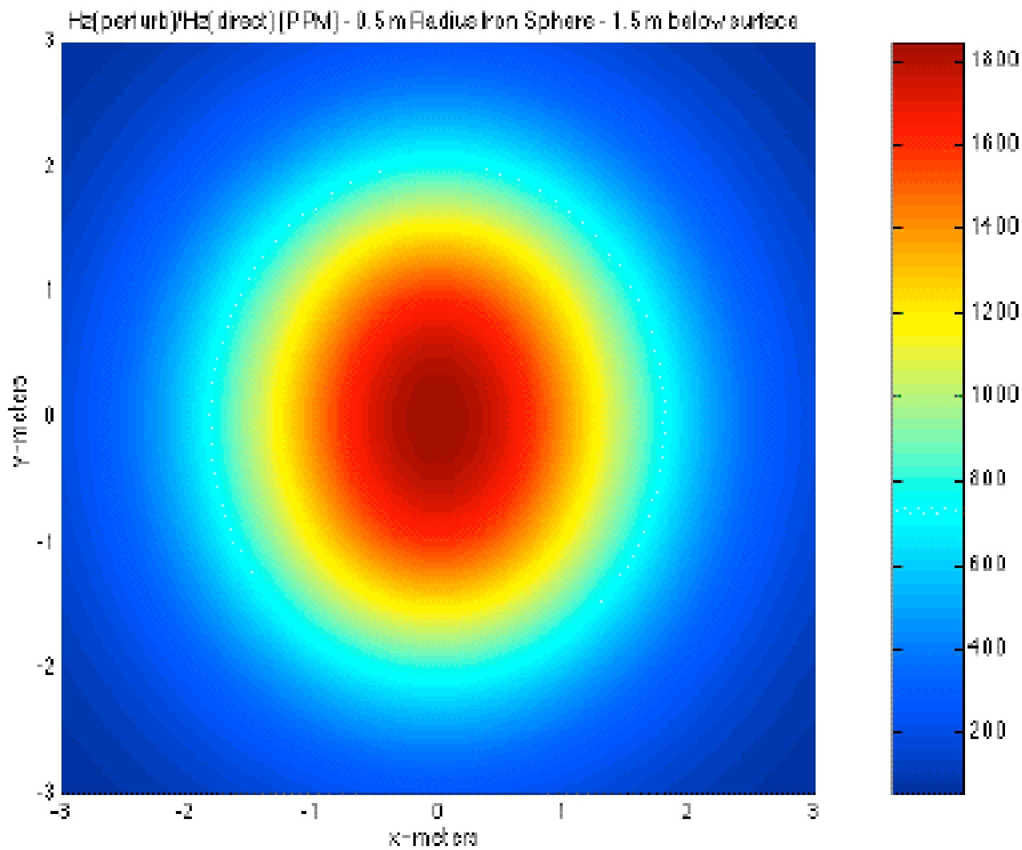
```
muSph   = muIron;
ax = 0.5;ay = 0.5;az = 0.5;

[inducedDipMomCoef] = ...
      getMagDipMomCond(freqVal,epsOut,muOut,epsSph,muSph,ax,ay,az);

RSourceUG  = REvaUG;
HFieldPertbZ  =  zeros(ny,nx);
for  iy=1:ny
      for  ix=1:nx
            gridPoint  =  [xx(1,ix),yy(iy,1),0];
            REvalPoints   = REvalPoints0   +   gridPoint;
            HFieldUG  =  [HFieldUGX(iy,ix),HFieldUGY(iy,ix),HFieldUGZ(iy,ix)];
            inducedDipMom  =  inducedDipMomCoef.*HFieldUG;
            [EFieldPertbTemp,HFieldPertbTemp]  =  ...
                  getMagDipoleField(EFieldOn,HFieldOn,inducedDipMom,...
                        RSourceUG,REvalPoints,freqVal,...
                        zLayer1,eDielec1,Conduct1,zLayer2,eDielec2,Conduct2);
            HFieldPertbZ(iy,ix)  =  HFieldPertbTemp(1,3);
      end
end

%   Find  absolute  Perturbed/Direct  (in  PPM)  for  H  Field  for  each  of  the
%   measurement  position  points  on  the  grid
HFieldRatioPPM  =  1.0E6.*abs(HFieldPertbZ./HFieldDirZ);

%     Plot  Perturbed  H  Field  in  PPM  for  each  of  the  measurement  position
%     points  on  the  grid
figure
pcolor(xx,yy,HFieldRatioPPM)
shading  interp
xlabel('x  -  meters')
ylabel('y  -  meters')
colorbar
titleStrL1  =  sprintf('Hz(perturb)/Hz(direct)  [PPM]\n');
titleStrL2  =  'A  0.5  m  Radius  Iron  Sphere  at  1.5  m  below  surface';
titleStr  =  [titleStrL1,titleStrL2];
title(titleStr);
axis  image
%***************************************************************************
```

We have supplied this MATLAB code, `LoopSensorExample.m`, along with `getMagDipMomCond.m` on the CD-ROM.

The results of this calculation are shown below as a color intensity plot of the ratio

$$| H_z^{Perturb} / H_z^{Direct} |$$

for the grid in PPM; the results have been smoothed using the MATLAB `shading interp` command.



| $H_z^{Perturb}$/ H_z^{Direct} | for buried sphere example - Color intensity scale in PPM

Menu Interface Routines

This section describes the MATLAB interface that allows a user to define the sources, receivers and buried structures. The user can then setup appropriate linkages to any computations to configure an easy to use menu driven interface. The menu interface software has been developed so that linkages to any user defined MATLAB code (including the Sommerfeld routines or the Hill/King buried wire routines) may be easily implemented by an experienced MATLAB user.

Objects

1. There are objects called "sources" and "receivers" that have locations that vary; i.e., they can move. These locations are called "measurement points" or "time steps". However, the order of the measurement points is not important; they are not time related. There may be any number, up to some maximum, of measurement points. There must be at least one measurement point.

There may be any number, up to some maximum, of the objects. For example, you may have 1 source and 3 receivers. All sources and receivers have the same number of measurement points. There must be at least one source and one receiver.

2. There are also objects known as "buried structures" that have locations that do not vary; i.e., they cannot move. Again there may be any number, up to some maximum, of these objects. And, there may be no (0) buried structures.

3. A location is specified in Cartesian coordinates as a point, (x, y, z), in meters referenced from an arbitrary origin, (0, 0, 0). A measurement point is also specified in meters, (x, y, z), from the same arbitrary origin.

4. All sources have a strength that is given in units appropriate to the type of the source. There are four types of sources: (1) loop, (2) electric dipole, (3) magnetic dipole and (4) straight wire.

a. A loop source is defined by its radius, in meters, and its orientation, the plane that is perpendicular to a specified unit vector located at its center point. The unit of strength of a loop source is "Amp". The location of the center of the loop will define the location of the loop. The loop intended here has finite size with respect to other dimensions of the problem, e.g. layer depths, distances to objects, etc.

b. An electric dipole source is located at a single point and is defined by its orientation, a specified unit vector at that point. The unit of strength of an electric dipole source is "Amp-m". It is an infinitesimal-length wire fed at its center, in terms of its radiated fields.
c. A magnetic dipole source is also located at a single point and defined by its orientation, a specified unit vector at that point. The unit of strength of a magnetic dipole source is "Amp-m^2". It is an infinitesimally small loop, whose axis coincides with the perpendicular to the loop center, with respect to its radiated fields.
d. A straight wire source is defined by its length and direction, both given by a specified vector with center at the midpoint of the wire. The location of the center of this vector will define the location of the wire. The unit of strength of a wire source is "Volt". If the wire length is short (approaching infinitesimal), the wire is usually better or more conveniently modeled by the electric dipole. The feed of the wire is assumed to be at the center (this may be changed by a MATLAB modification to the menu interface program).

5. There are four types of receivers, the same as the sources. However, receivers do not have an associated strength. All other properties are the same as the four corresponding types of sources.

6. There are two types of buried structures: (1) ellipsoid and (2) wire.
a. An ellipsoid buried structure is defined by its 3 semi-axes, ax, ay and az; its orientation, which may be specified by two unit vectors (one corresponding to its local z-axis and one corresponding to its local x-axis) or three rotations (angular rotations about each of its local axes); and its electrical properties which are its relative permeability, relative dielectric constant or permittivity, and whether or not it is a perfect conductor. The center of the ellipsoid will define its location.
b. A straight wire buried structure is defined by its length and direction, both given by a specified vector with center at the midpoint of the wire. The center of this vector will define its location.

Global Variables
The above information is communicated to the rest of the program via the following global variables, found in the m-file "SRLGlobals.m".

Sources/Receivers

nMeasure - the number of measurement points.
nSource - the number of sources.
nReceiver - the number of receivers.

xSource, ySource, zSource - arrays (nMeasure,nSource).
These arrays contain the measurement locations in meters of all sources.

sourceType - array (1,nSource).
This array contains the type of each source, 1 == loop, 2 == electric dipole, 3 == magnetic dipole and 4 == wire.

sourceStrength - array (1,nSource).
This array contains the strength of each source.

sourceProperties - array (nSource,4).
This array contains the defining properties of each source. There is one row per source and the contents of each row varies with the type of source as follows:

loop - columns 1-3 == the orientation unit vector, column 4 == radius of the loop in meters;
dipoles - columns 1-3 == the orientation unit vector, column 4 == not used;
wire - columns 1-3 == the orientation and length vector, column 4 == not used;

xReceiver, yReceiver, zReceiver - arrays(nMeasure,nReceiver).
These arrays contain the measurement locations in meters of all receivers.

receiverType - array (1,nReceiver).
This array is similar to the sourceType array.

receiverProperties - array (nReceiver,4).
This array is similar to the sourceProperties array.

Additional source/receiver variables

The following variables are created if the user decides to have measurement locations computed from a grid.

xGrid, yGrid, zGrid - arrays (4).

These arrays contain the definition of the computed grid. The four elements of each array define the minimum value in meters, the maximum value in meters, the number of grid points and the distance between grid points in meters - for each axis.

x0Source, y0Source, z0Source - arrays(nSource).

These arrays contain the location of each source relative to the origin of the grid.

x0Receiver, y0Receiver, z0Receiver - arrays(nReceiver).

These arrays contain the location of each receiver relative to the origin of the grid.

Buried Structures (Ellipsoids)

nElip - the number of ellipsoids defined.

xElip, yElip, zElip - arrays (1,nElip).

These arrays give the locations, in meters, of the centers of the various buried ellipsoids.

ixElip, iyElip, izElip - arrays (1,nElip).
jxElip, jyElip, jzElip - arrays (1,nElip).
kxElip, kyElip, kzElip - arrays (1,nElip).

These arrays describe three unit vectors for each ellipsoid. The unit vectors give the orientation of each ellipsoid. The unit vector (jxElip, jyElip, jzElip) is computed by the program given the other two.

dxElip, dyElip, dzElip - arrays (1,nElip).

These arrays give the lengths, in meters, of each of the three semi-axes of each ellipsoid.

pcElip - array (1,nElip).

This array provides a "perfect conductor" flag for each ellipsoid. (== 0 means the ellipsoid is not a perfect conductor).

epsElip - complex array (1,nElip).

This array provides the relative complex dielectric constant for each ellipsoid.

muElip - complex array (1,nElip).

This array provides the relative complex permeability of each ellipsoid.

Buried Structures (Wires)

nWire - the number of wires defined.

xWire, yWire, zWire - arrays (1,nWire).

These arrays give the locations, in meters, of the centers of the various buried wires.

vxWire, vyWire, vzWire - arrays (1,nWire).

These arrays describe a vector for each wire. The center of the vector is located at the center of its corresponding wire. And the direction of the vector is the direction of the wire. And the length of the vector is the length of the wire (meters).

Private Global Variables

The source/receiver and buried structures information is organized into arrays of structures for the use of the input portion of the program. These arrays of structures are defined within a "private" global area. (See the m-file "SRLPrivates.m".) Here are descriptions of these private global variables.

Sources/Receivers

SRLmethod - the method of input, 0 == by computation, 1 == by file.

SRLpathname - the path and name of the input file.

This variable is used if the input is by file.

SRLgridX, SRLgridY, SRLgridZ

These arrays hold the same information as the variables xGrid, yGrid and zGrid described above.

SRLSn - the number of sources.

SRLSmax - the maximum allowed number of sources.

SRLSrc - An array of structures (a cell array).

Each structure within this array contains the relative measurement location and defining properties of one source. There is one structure per source. The cell array is preallocated with the MATLAB statement "SRLSrc = cell(SRLSmax,1);".

Here are the elements of each structure:

.type = integer - the type of source,

1 == loop

2 == electric dipole

3 == magnetic dipole

4 == wire

.location = array (1,3)

The location (x,y,z - meters) of this source relative to the current measurement point

.vector = array (1,3) the orientation (and/or length) vector

.radius = the radius of a loop source.

.strength = complex - the source strength;

Specific quantities within the array of structures are referred to with notation in the form of "SRLSrc{5}.location(2)", which refers to the y-coordinate of the relative location of the 5th source.

SRLRn - the number of receivers.

SRLRmax - the maximum allowed number of receivers.

SRLRcv - this is a cell array of structures that define the locations and properties of the receivers. It is defined similar to the above cell array. The only difference is that the value of the "strength" member of the structures within the array will be set to zero.

Buried Structures

BSDpathname - the path and name of the input file.
This variable is used if the input is by file.

BSDEn - the number of ellipsoids.

BSDEmax - the maximum allowed number of ellipsoids.

BSDEllip - cell array of ellipsoid buried structures
One structure per ellipsoid defined as follows:
.location = array (1,3), the location of the center of the ellipsoid in Cartesian coordinates (x,y,z - meters)
.axes = array (1,3), the lengths of the semi-axis, x, y and z, in meters
.xunit= array (1,3), unit vector describing the orientation of the ellipsoid's x-axis relative to its local origin
.yunit= array (1,3), unit vector describing the orientation of the ellipsoid's y-axis (this vector is calculated)
.zunit= array (1,3), unit vector describing the orientation of the ellipsoid's z-axis relative to its local origin
.angles = array (1,3), alternate orientation expressed as angles of rotation about the respective axes (degrees), (the x,y,z-unit vectors are computed from this),
.orient = integer, orientation flag, 1==unit vectors, 0==angles,
.pc = integer, perfect conductor flag, 1==yes, 0==no,
.eps= complex, relative dielectric constant,
.mu = complex, relative permeability constant.

BSDWn - the number of wires.

BSDWmax - the maximum allowed number of wires.

BSDWire - cell array of wire buried structures
One structure per wire defined as follows:
.location = array(1,3), the location of the midpoint of the wire in Cartesian coordinates (x,y,z - meters)
.vector = array(1,3), a vector with its center at the midpoint of the wire. This vector describes length (meters) and direction.

Operation

To input source and receiver locations invoke the MATLAB function "SRLocations". To input the buried structures definitions invoke the MATLAB function "BSDefinition". Each of these functions will present a dialog box that allows the user to select between input by computation or input by file.

Input via File

If you intend to select the input by file method you must have on the disk a file that is readable via the MATLAB "load filename" statement.

If your file input is for the sources and receivers then your input file must have all of the source and receivers variables described in the sections "Sources/Receivers" and "Additional source/receiver variables" in the "Globals" section above.

If your file input is for the buried structures then your input file must have all of the buried structures' variables described in the sections "Buried Structures (Ellipsoids)" and "Buried Structures (Wires)" in the "Globals" section above.

Source/Receiver Input via Computation

When you input your source/receiver definitions via computation you must define the grid of measurement locations, define at least one source and define at least one receiver. At the conclusion of your input the definitions will be converted and copied to the global variables described in the "Globals" section above. These values will also be written to the file "SRLSave.mat" in a form that is readable via the MATLAB "load" statement.

Each time the "SRLocations" function is invoked it will look for the "SRLSave.mat" file and read it and present those values to you when you select input by computation. If you want to start from scratch discard the file "SRLSave.mat".

The dialog boxes presented during the input by computation are self explanatory. Each box represents one source or one receiver. You navigate from source to source or receiver to receiver via the buttons within the box. You can choose to add or delete items. When you delete an item it is moved to the end of the list and the list is shortened by one. Thus, when you subsequently add a new item, the one you deleted appears at the end of the list for user information.

Buried Structures Input via Computation

When you input your buried structures definitions via computation the process is similar to that described above. At the conclusion of your input the definitions will be converted and copied to the global variables described in the "Globals" section above. And these values will also be written to the file "BSDSave.mat" in a form that is readable via the MATLAB "load" statement.

Each time the "BSDefinition" function is invoked it will look for the "BSDSave.mat" file and read it and present those values to you when you select input by computation. And, if you want to start from scratch then discard the file "BSDSave.mat".

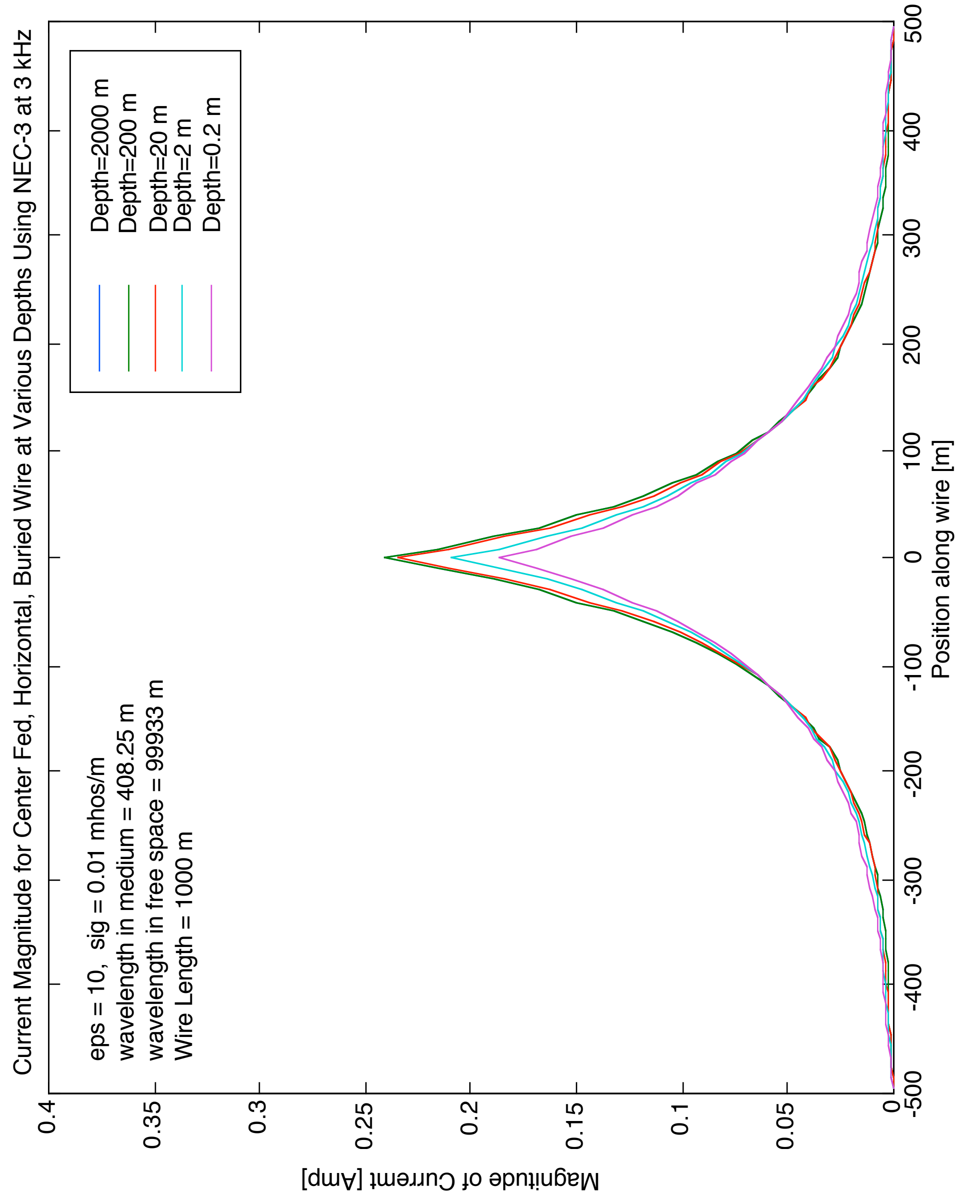
The dialog boxes presented for buried structure definitions work the same as those for source/receiver definitions. There is one box for each item. You may have zero items, in which case all entries in the box are disabled except the "add" button.

Hill/King Buried Wire Routines

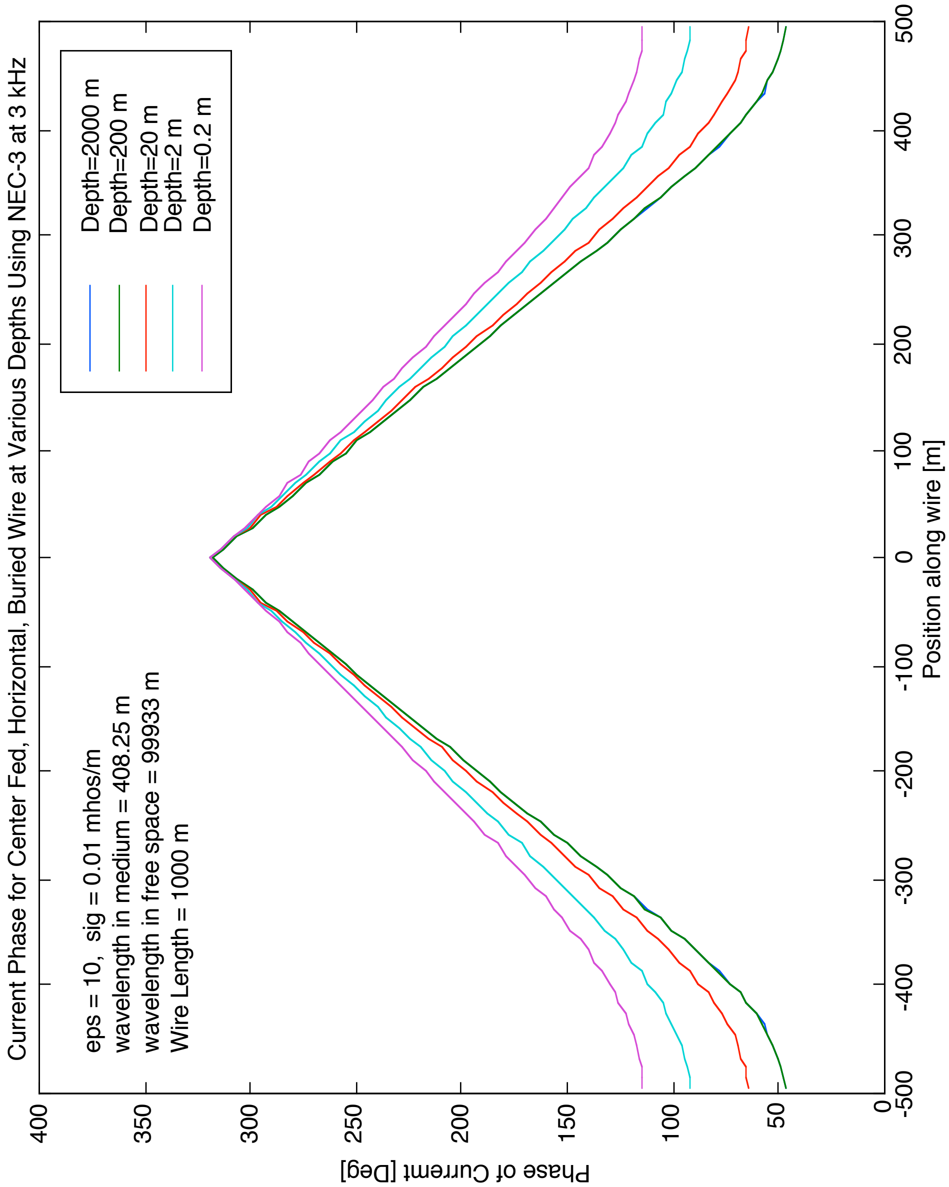
For a number of applications it is useful to model long insulated conductor in a lossy medium with arbitrary termination on either end. We have written MATLAB routines that use David A. Hill's paper (expanding on previous work of R.W.P. King - "Magnetic Dipole Excitation of an Insulated Conductor of Finite Length", IEEE Trans. Geosci. Remote Sensing, vol. 28, pp. 289-294,1990) to do these calculations. The Hill/King result allows current on an insulated conductor in a lossy medium with arbitrary termination on either end to be obtained from a defined, imposed, external electric field.

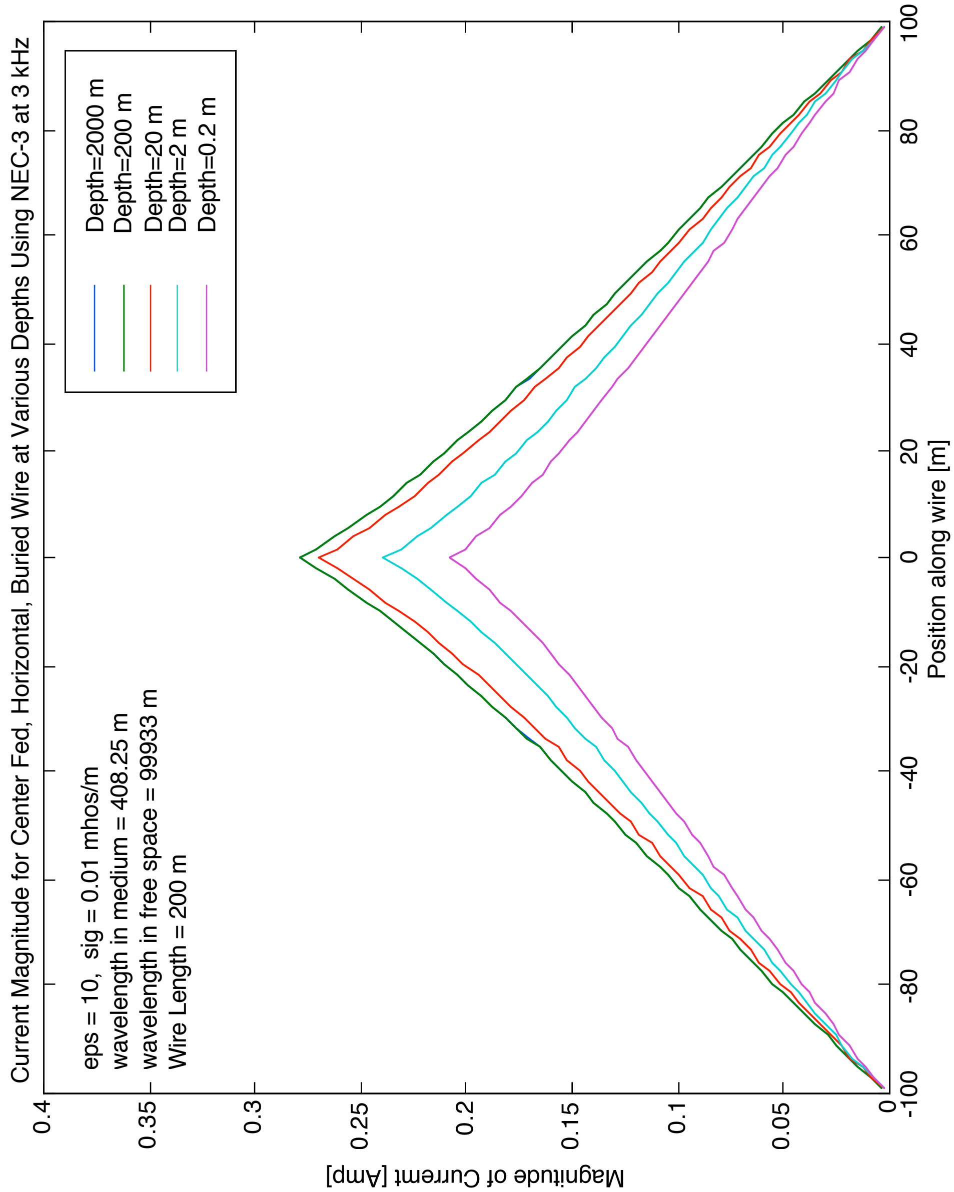
The Hill/King model assumes that the wire is buried in a single infinite medium; we are interested in multilayered applications - so there is a issue of how the current is affected by getting near a medium interface; we have used NEC-3 to address this issue (discussed in detail below). It is our opinion that the single infinite medium approximation for the calculation of the Hill/King current will work well for multilayer applications. Included on the CD-ROM are examples using this code; the examples are taken from Hill's paper reproducing Figures 6-10 in the paper.

To determine the effect of a medium interface on a current distribution we used NEC-3 to look at the current distribution as a voltage excited horizontal buried wire moves closer to a medium interface. We used 3 kHz and average earth (ϵ=10, σ=0.01 mhos/m). We looked at two lengths of wire: one a little under 2.5 λ_{medium} in length (1000 m) and one a little under 0.5 λ_{medium} in length (200 m). The wire was fed at the center with 1 Volt. The depths used were 2000 m, 200 m, 20 m, 2 m, and 0.2 m. The results of these calculations show that there is a small change in the magnitude of the current and a slight increase in the phase as the wire approaches the interface.

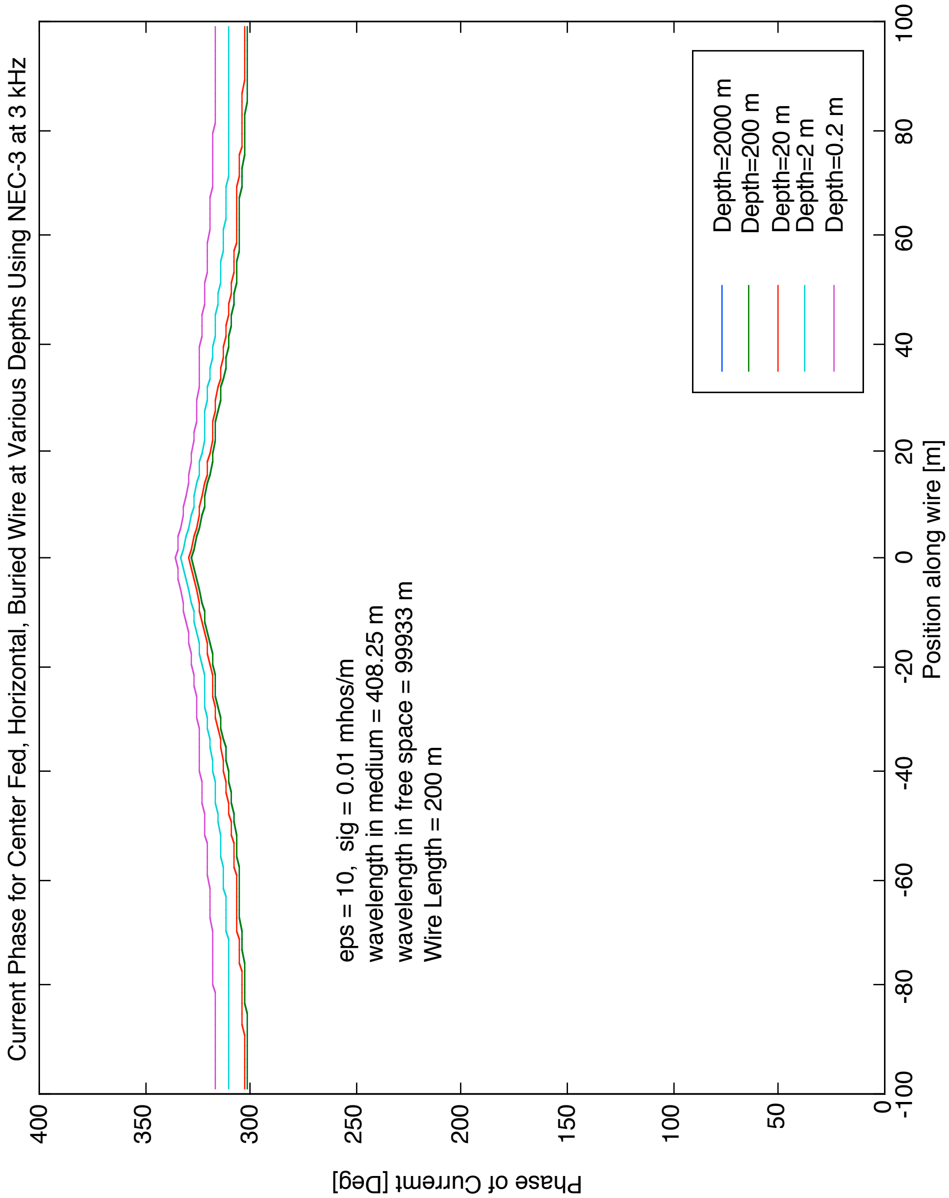


Current Magnitude for Center Fed, Horizontal, Buried Wire at Various Depths Using NEC-3 at 3 kHz
eps = 10, sig = 0.01 mhos/m
wavelength in medium = 408.25 m
wavelength in free space = 99933 m
Wire Length = 1000 m
Depth=2000 m
Depth=200 m
Depth=20 m
Depth=2 m
Depth=0.2 m
Magnitude of Curremt [Amp]
Position along wire [m]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
-500
-400
-300
-200
-100
0
100
200
300
400
500


Current Phase for Center Fed, Horizontal, Buried Wire at Various Depths Using NEC-3 at 3 kHz
eps = 10, sig = 0.01 mhos/m
wavelength in medium = 408.25 m
wavelength in free space = 99933 m
Wire Length = 1000 m
Depth=2000 m
Depth=200 m
Depth=20 m
Depth=2 m
Depth=0.2 m
Phase of Current [Deg]
Position along wire [m]



Current Magnitude for Center Fed, Horizontal, Buried Wire at Various Depths Using NEC-3 at 3 kHz
eps = 10, sig = 0.01 mhos/m
wavelength in medium = 408.25 m
wavelength in free space = 99933 m
Wire Length = 200 m
Depth=2000 m
Depth=200 m
Depth=20 m
Depth=2 m
Depth=0.2 m
Magnitude of Curremt [Amp]
Position along wire [m]


Current Phase for Center Fed, Horizontal, Buried Wire at Various Depths Using NEC-3 at 3 kHz
eps = 10, sig = 0.01 mhos/m
wavelength in medium = 408.25 m
wavelength in free space = 99933 m
Wire Length = 200 m
Depth=2000 m
Depth=200 m
Depth=20 m
Depth=2 m
Depth=0.2 m
Phase of Current [Deg]
Position along wire [m]
0
50
100
150
200
250
300
350
400
-100
-80
-60
-40
-20
0
20
40
60
80
100

Appendix A - Sommerfeld Integral Definitions

Definitions: $k_{nz} = \sqrt{k_n^2 - k_\rho^2}$, $k_n = \omega\sqrt{\mu_n \varepsilon_n}$

$$T_{12}^{TM} = \frac{2\varepsilon_2 k_{1z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}, \quad T_{12}^{TE} = \frac{2\mu_2 k_{1z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}, \quad T_{21}^{TM} = \frac{2\varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}, \quad T_{21}^{TE} = \frac{2\mu_1 k_{2z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}$$

$$R_{12}^{TM} = \frac{\varepsilon_2 k_{1z} - \varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}, \quad R_{12}^{TE} = \frac{\mu_2 k_{1z} - \mu_1 k_{2z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}, \quad R_{21}^{TM} = -R_{12}^{TM}, \quad R_{21}^{TE} = -R_{12}^{TE}$$

$$T_{23}^{TM} = \frac{2\varepsilon_3 k_{2z}}{\varepsilon_3 k_{2z} + \varepsilon_2 k_{3z}}, \quad T_{23}^{TE} = \frac{2\mu_3 k_{2z}}{\mu_3 k_{2z} + \mu_2 k_{3z}}, \quad R_{23}^{TM} = \frac{\varepsilon_3 k_{2z} - \varepsilon_2 k_{3z}}{\varepsilon_3 k_{2z} + \varepsilon_2 k_{3z}}, \quad R_{23}^{TE} = \frac{\mu_3 k_{2z} - \mu_2 k_{3z}}{\mu_3 k_{2z} + \mu_2 k_{3z}}$$

$$\tilde{R}_{12}^{TM} = \frac{R_{12}^{TM} + \tilde{R}_{23}^{TM} e^{2ik_{2z}(d_2 - d_1)}}{1 + R_{12}^{TM}\tilde{R}_{23}^{TM} e^{2ik_{2z}(d_2 - d_1)}}, \qquad \tilde{R}_{12}^{TE} = \frac{R_{12}^{TE} + \tilde{R}_{23}^{TE} e^{2ik_{2z}(d_2 - d_1)}}{1 + R_{12}^{TE}\tilde{R}_{23}^{TE} e^{2ik_{2z}(d_2 - d_1)}}$$

$$\tilde{R}_{21}^{TM} = R_{21}^{TM}, \quad \tilde{R}_{23}^{TM} = R_{23}^{TM}, \quad \tilde{R}_{21}^{TE} = R_{21}^{TE}, \quad \tilde{R}_{23}^{TE} = R_{23}^{TE}$$

$$\tilde{T}_{21}^{TM} = T_{21}^{TM} = \frac{\varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z}} T_{12}^{TM}, \qquad \tilde{T}_{21}^{TE} = T_{21}^{TE} = \frac{\mu_1 k_{2z}}{\mu_2 k_{1z}} T_{12}^{TE}$$

$$A_2^{TM} = \frac{T_{12}^{TM} e^{i(k_{1z} - k_{2z})d_1}}{1 + R_{12}^{TM}\tilde{R}_{23}^{TM} e^{2ik_{2z}(d_2 - d_1)}}, \qquad A_2^{TE} = \frac{T_{12}^{TE} e^{i(k_{1z} - k_{2z})d_1}}{1 + R_{12}^{TE}\tilde{R}_{23}^{TE} e^{2ik_{2z}(d_2 - d_1)}}$$

$$B_2^{TM} = \frac{e^{ik_{2z}z_0} - R_{23}^{TM} e^{ik_{2z}(2d_2 - z_0)}}{1 + R_{12}^{TM} R_{23}^{TM} e^{2ik_{2z}(d_2 - d_1)}}, \qquad B_2^{TE} = \frac{e^{ik_{2z}z_0} + e^{ik_{2z}(2d_2 - z_0)} R_{23}^{TE}}{1 + R_{12}^{TE}\tilde{R}_{23}^{TE} e^{2ik_{2z}(d_2 - d_1)}}$$

$$C_2^{TM} = \frac{-e^{-ik_{2z}z_0} + e^{-ik_{2z}(2d_1 - z_0)} R_{21}^{TM}}{1 + R_{12}^{TM}\tilde{R}_{23}^{TM} e^{2ik_{2z}(d_2 - d_1)}}, \qquad C_2^{TE} = \frac{e^{-ik_{2z}z_0} + e^{-ik_{2z}(2d_1 - z_0)} R_{21}^{TE}}{1 + R_{12}^{TE}\tilde{R}_{23}^{TE} e^{2ik_{2z}(d_2 - d_1)}}$$

$$M_2^{TM} = \frac{e^{ik_{2z}z_0} + R_{23}^{TM} e^{ik_{2z}(2d_2 - z_0)}}{1 + R_{12}^{TM} R_{23}^{TM} e^{2ik_{2z}(d_2 - d_1)}}, \qquad M_2^{TE} = \frac{e^{ik_{2z}z_0} - e^{ik_{2z}(2d_2 - z_0)} R_{23}^{TE}}{1 + R_{12}^{TE}\tilde{R}_{23}^{TE} e^{2ik_{2z}(d_2 - d_1)}}$$

$$N_2^{TM} = \frac{e^{-ik_{2z}z_0} + e^{-ik_{2z}(2d_1 - z_0)} R_{21}^{TM}}{1 + R_{12}^{TM}\tilde{R}_{23}^{TM} e^{2ik_{2z}(d_2 - d_1)}}, \qquad N_2^{TE} = \frac{-e^{-ik_{2z}z_0} + e^{-ik_{2z}(2d_1 - z_0)} R_{21}^{TE}}{1 + R_{12}^{TE}\tilde{R}_{23}^{TE} e^{2ik_{2z}(d_2 - d_1)}}$$

VED in Region 1

Located at $z=0$ - Interface 1-2 at $z=-d_1$ - Interface 2-3 at $z=-d_2$

Fields in Region 1: ($z \geq -d_1$)

$$E_{1\phi}^{VED[1]}=0 \quad , \quad H_{1\rho}^{VED[1]}=0 \quad , \quad H_{1z}^{VED[1]}=0$$

$$E_{1z}^{VED[1]}=-\frac{Il}{4\pi\omega\varepsilon_1}\int_0^\infty dk_\rho \frac{k_\rho^3}{k_{1z}} J_0(k_\rho \rho)\left[e^{ik_{1z}|z|}+\tilde{R}_{12}^{TM}e^{ik_{1z}(z+2d_1)}\right]$$

$$E_{1\rho}^{VED[1]}=i\frac{Il}{4\pi\omega\varepsilon_1}\begin{cases}\int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho\rho)\left[e^{ik_{1z}|z|}+\tilde{R}_{12}^{TM}e^{ik_{1z}(z+2d_1)}\right] & z\geq 0\\ \int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho\rho)\left[-e^{ik_{1z}|z|}+\tilde{R}_{12}^{TM}e^{ik_{1z}(z+2d_1)}\right] & -d_1 \leq z<0\end{cases}$$

$$H_{1\phi}^{VED[1]}=i\frac{Il}{4\pi}\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{1z}} J_1(k_\rho\rho)\left[e^{ik_{1z}|z|}+\tilde{R}_{12}^{TM}e^{ik_{1z}(z+2d_1)}\right]$$

Fields in Region 2: ($-d_1 > z \geq -d_2$)

$$E_{2\phi}^{VED[1]}=0 \quad , \quad H_{2\rho}^{VED[1]}=0 \quad , \quad H_{2z}^{VED[1]}=0$$

$$E_{2z}^{VED[1]}=-\frac{Il}{4\pi\omega\varepsilon_2}\int_0^\infty dk_\rho \frac{k_\rho^3}{k_{1z}} J_0(k_\rho\rho)A_2^{TM}\left[e^{-ik_{2z}z}+\tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right]$$

$$E_{2\rho}^{VED[1]}=i\frac{Il}{4\pi\omega\varepsilon_2}\int_0^\infty dk_\rho \frac{k_{2z}k_\rho^2}{k_{1z}} J_1(k_\rho\rho)A_2^{TM}\left[-e^{-ik_{2z}z}+\tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right]$$

$$H_{2\phi}^{VED[1]}=i\frac{Il}{4\pi}\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{1z}} J_1(k_\rho\rho)A_2^{TM}\left[e^{-ik_{2z}z}+\tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right]$$

VMD in Region 1

Located at $z = 0$ - Interface 1-2 at $z = -d_1$ - Interface 2-3 at $z = -d_2$

Fields in Region 1: ($z \geq -d_1$)

$$E_{1z}^{VMD[1]} = 0 \quad , \quad E_{1\rho}^{VMD[1]} = 0 \quad , \quad H_{1\phi}^{VMD[1]} = 0$$

$$E_{1\phi}^{VMD[1]} = \frac{-\omega\mu_1 IA}{4\pi} \int_0^\infty dk_\rho \frac{k_\rho^2}{k_{1z}} J_1(k_\rho \rho)\left[e^{ik_{1z}|z|} + \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right]$$

$$H_{1z}^{VMD[1]} = -i\frac{IA}{4\pi} \int_0^\infty dk_\rho \frac{k_\rho^3}{k_{1z}} J_0(k_\rho \rho)\left[e^{ik_{1z}|z|} + \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right]$$

$$H_{1\rho}^{VMD[1]} = -\frac{IA}{4\pi} \begin{cases} \int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho \rho)\left[e^{ik_{1z}|z|} + \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right] & z \geq 0 \\ \int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho \rho)\left[-e^{ik_{1z}|z|} + \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right] & -d_1 \leq z < 0 \end{cases}$$

Fields in Region 2: ($-d_1 > z \geq -d_2$)

$$E_{2z}^{VMD[1]} = 0 \quad , \quad E_{2\rho}^{VMD[1]} = 0 \quad , \quad H_{2\phi}^{VMD[1]} = 0$$

$$E_{2\phi}^{VMD[1]} = -\omega\mu_1 \frac{IA}{4\pi} \int_0^\infty dk_\rho \frac{k_\rho^2}{k_{1z}} J_1(k_\rho \rho) A_2^{TE}\left[e^{-ik_{2z}z} + \tilde{R}_{23}^{TE} e^{ik_{2z}(z+2d_2)}\right]$$

$$H_{2z}^{VMD[1]} = -i\frac{IA\mu_1}{4\pi\mu_2} \int_0^\infty dk_\rho \frac{k_\rho^3}{k_{1z}} J_0(k_\rho \rho) A_2^{TE}\left[e^{-ik_{2z}z} + \tilde{R}_{23}^{TE} e^{ik_{2z}(z+2d_2)}\right]$$

$$H_{2\rho}^{VMD[1]} = -\frac{IA\mu_1}{4\pi\mu_2} \int_0^\infty dk_\rho \frac{k_{2z}k_\rho^2}{k_{1z}} J_1(k_\rho \rho) A_2^{TE}\left[-e^{-ik_{2z}z} + \tilde{R}_{23}^{TE} e^{ik_{2z}(z+2d_2)}\right]$$

VED in Region 2

Located at $z=-z_0\ (-d_2 < -z_0 < -d_1)$ - Interface 1-2 at $z=-d_1$ - Interface 2-3 at $z=-d_2$

Fields in Region 1: ($z \geq -d_1$)

$$E_{1\phi}^{VED[2]}=0\ ,\quad H_{1\rho}^{VED[2]}=0\ ,\quad H_{1z}^{VED[2]}=0$$

$$E_{1z}^{VED[2]}=-\frac{Il}{4\pi\omega\varepsilon_1}\int_0^\infty dk_\rho \frac{k_\rho^3}{k_{2z}} J_0\left(k_\rho\rho\right) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

$$E_{1\rho}^{VED[2]}=i\frac{Il}{4\pi\omega\varepsilon_1}\int_0^\infty dk_\rho \frac{k_{1z}k_\rho^2}{k_{2z}} J_1\left(k_\rho\rho\right) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

$$H_{1\phi}^{VED[2]}=i\frac{Il}{4\pi}\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{2z}} J_1\left(k_\rho\rho\right) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

Fields in Region 2 <u>above the source</u>: ($-d_1 > z \geq -z_0$)

$$E_{2\phi}^{VED[2]}=0\ ,\quad H_{2\rho}^{VED[2]}=0\ ,\quad H_{2z}^{VED[2]}=0$$

$$E_{2z}^{VED[2]}=-\frac{Il}{4\pi\omega\varepsilon_2}\int_0^\infty dk_\rho \frac{k_\rho^3}{k_{2z}} J_0\left(k_\rho\rho\right) M_2^{TM}\left(e^{ik_{2z}z}+e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$E_{2\rho}^{VED[2]}=i\frac{Il}{4\pi\omega\varepsilon_2}\int_0^\infty dk_\rho k_\rho^2 J_1\left(k_\rho\rho\right) M_2^{TM}\left(e^{ik_{2z}z}-e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$H_{2\phi}^{VED[2]}=i\frac{Il}{4\pi}\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{2z}} J_1\left(k_\rho\rho\right) M_2^{TM}\left(e^{ik_{2z}z}+e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

VMD in Region 2

Located at $z = -z_0$ ($-d_2 < -z_0 < -d_1$) - Interface 1-2 at $z = -d_1$ - Interface 2-3 at $z = -d_2$

Fields in Region 1: ($z \geq -d_1$)

$$H_{1\phi}^{VMD[2]} = 0 \quad , \quad E_{1\rho}^{VMD[2]} = 0 \quad , \quad E_{1z}^{VMD[2]} = 0$$

$$H_{1z}^{VMD[2]} = -i \frac{IA\mu_2}{4\pi\mu_1} \int_0^\infty dk_\rho \frac{k_\rho^3}{k_{2z}} J_0(k_\rho \rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1} \tilde{T}_{21}^{TE} M_2^{TE}$$

$$H_{1\rho}^{VMD[2]} = -\frac{IA\mu_2}{4\pi\mu_1} \int_0^\infty dk_\rho \frac{k_{1z}k_\rho^2}{k_{2z}} J_1(k_\rho \rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1} \tilde{T}_{21}^{TE} M_2^{TE}$$

$$E_{1\phi}^{VMD[2]} = \omega\mu_2 \frac{IA}{4\pi} \int_0^\infty dk_\rho \frac{k_\rho^2}{k_{2z}} J_1(k_\rho \rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1} \tilde{T}_{21}^{TE} M_2^{TE}$$

Fields in Region 2 <u>above the source</u>: ($-d_1 > z \geq -z_0$)

$$H_{2\phi}^{VMD[2]} = 0 \quad , \quad E_{2\rho}^{VMD[2]} = 0 \quad , \quad E_{2z}^{VMD[2]} = 0$$

$$H_{2z}^{VMD[2]} = -i \frac{IA}{4\pi} \int_0^\infty dk_\rho \frac{k_\rho^3}{k_{2z}} J_0(k_\rho \rho) M_2^{TE} \left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)} R_{21}^{TE}\right)$$

$$H_{2\rho}^{VMD[2]} = -\frac{IA}{4\pi} \int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho \rho) M_2^{TE} \left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)} R_{21}^{TE}\right)$$

$$E_{2\phi}^{VMD[2]} = \omega\mu_2 \frac{IA}{4\pi} \int_0^\infty dk_\rho \frac{k_\rho^2}{k_{2z}} J_1(k_\rho \rho) M_2^{TE} \left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)} R_{21}^{TE}\right)$$

HED in Region 1

Located at $z = 0$ - Interface 1-2 at $z = -d_1$ - Interface 2-3 at $z = -d_2$

Fields in Region 1: ($z \geq -d_1$)

$$E_{1z}^{HED[1]} = i\frac{Il}{4\pi\omega\varepsilon_1}\cos\phi \begin{cases} \int_0^\infty dk_\rho k_\rho^2 J_1\left(k_\rho\rho\right)\left(e^{ik_{1z}|z|} - \tilde{R}_{12}^{TM} e^{ik_{1z}(z+2d_1)}\right) & z \geq 0 \\ \int_0^\infty dk_\rho k_\rho^2 J_1\left(k_\rho\rho\right)\left(-e^{ik_{1z}|z|} - \tilde{R}_{12}^{TM} e^{ik_{1z}(z+2d_1)}\right) & z < 0 \end{cases}$$

$$H_{1z}^{HED[1]} = i\frac{Il}{4\pi}\sin\phi\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{1z}} J_1\left(k_\rho\rho\right)\left(e^{ik_{1z}|z|} + \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right)$$

$$E_{1\rho}^{HED[1]} = -\frac{Il}{4\pi\omega\varepsilon_1}\cos\phi\int_0^\infty dk_\rho k_{1z} k_\rho \left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{ik_{1z}|z|} - \tilde{R}_{12}^{TM} e^{ik_{1z}(z+2d_1)}\right)$$

$$-k_1^2\frac{Il}{4\pi\omega\varepsilon_1\rho}\cos\phi\int_0^\infty dk_\rho \frac{1}{k_{1z}} J_1\left(k_\rho\rho\right)\left(e^{ik_{1z}|z|} + \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right)$$

$$E_{1\phi}^{HED[1]} = \frac{Il}{4\pi\omega\varepsilon_1\rho}\sin\phi\int_0^\infty dk_\rho k_{1z} J_1\left(k_\rho\rho\right)\left(e^{ik_{1z}|z|} - \tilde{R}_{12}^{TM} e^{ik_{1z}(z+2d_1)}\right)$$

$$+k_1^2\frac{Il}{4\pi\omega\varepsilon_1}\sin\phi\int_0^\infty dk_\rho \frac{1}{k_{1z}} k_\rho \left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{ik_{1z}|z|} + \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right)$$

$$H_{1\rho}^{HED[1]} = \frac{Il}{4\pi}\sin\phi\int_0^\infty dk_\rho k_\rho \left[\frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\left(\tilde{R}_{12}^{TE} + \tilde{R}_{12}^{TM}\right) - J_0\left(k_\rho\rho\right)\tilde{R}_{12}^{TE}\right] e^{ik_{1z}(z+2d_1)}$$

$$+\frac{Il}{4\pi}\sin\phi \begin{cases} -\int_0^\infty dk_\rho k_\rho J_0\left(k_\rho\rho\right) e^{ik_{1z}|z|} & z \geq 0 \\ \int_0^\infty dk_\rho k_\rho J_0\left(k_\rho\rho\right) e^{ik_{1z}|z|} & -d_1 \leq z < 0 \end{cases}$$

$$H_{1\phi}^{HED[1]} = \frac{Il}{4\pi}\cos\phi\int_0^\infty dk_\rho k_\rho \left[J_0\left(k_\rho\rho\right)\tilde{R}_{12}^{TM} - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\left(\tilde{R}_{12}^{TM} + \tilde{R}_{12}^{TE}\right)\right] e^{ik_{1z}(z+2d_1)}$$

$$+\frac{Il}{4\pi}\cos\phi \begin{cases} -\int_0^\infty dk_\rho k_\rho J_0\left(k_\rho\rho\right) e^{ik_{1z}|z|} & z \geq 0 \\ \int_0^\infty dk_\rho k_\rho J_0\left(k_\rho\rho\right) e^{ik_{1z}|z|} & -d_1 \leq z < 0 \end{cases}$$

Fields in Region 2: ($-d_1 > z \geq -d_2$)

$$E_{2z}^{HED[1]} = -i\frac{Il}{4\pi\omega\varepsilon_2}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho\rho)A_2^{TM}\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$H_{2z}^{HED[1]} = i\frac{Il}{4\pi}\frac{\mu_1}{\mu_2}\sin\phi\int_0^\infty dk_\rho\frac{k_\rho^2}{k_{1z}}J_1(k_\rho\rho)A_2^{TE}\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$E_{2\rho}^{HED[1]} = \frac{Il}{4\pi\omega\varepsilon_2}\cos\phi\int_0^\infty dk_\rho A_2^{TM}k_{2z}k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(-e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$-k_2^2\frac{Il}{4\pi\omega\rho\varepsilon_2}\frac{\mu_1}{\mu_2}\cos\phi\int_0^\infty dk_\rho A_2^{TE}\frac{1}{k_{1z}}J_1(k_\rho\rho)\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$E_{2\phi}^{HED[1]} = \frac{Il}{4\pi\omega\rho\varepsilon_2}\sin\phi\int_0^\infty dk_\rho A_2^{TM}k_{2z}J_1(k_\rho\rho)\left(e^{-ik_{2z}z} - \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$+k_2^2\frac{Il}{4\pi\omega\varepsilon_2}\frac{\mu_1}{\mu_2}\sin\phi\int_0^\infty dk_\rho\frac{A_2^{TE}k_\rho}{k_{1z}}\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$H_{2\rho}^{HED[1]} = \frac{Il}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho A_2^{TM}J_1(k_\rho\rho)\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$-\frac{Il}{4\pi}\frac{\mu_1}{\mu_2}\sin\phi\int_0^\infty dk_\rho\frac{A_2^{TE}k_{2z}k_\rho}{k_{1z}}\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(-e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$H_{2\phi}^{HED[1]} = \frac{Il}{4\pi}\cos\phi\int_0^\infty dk_\rho A_2^{TM}k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$-\frac{Il}{4\pi\rho}\frac{\mu_1}{\mu_2}\cos\phi\int_0^\infty dk_\rho\frac{A_2^{TE}k_{2z}}{k_{1z}}J_1(k_\rho\rho)\left(-e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

HMD in Region 1

Located at $z = 0$ - Interface 1-2 at $z = -d_1$ - Interface 2-3 at $z = -d_2$

Fields in Region 1: ($z \geq -d_1$)

$$H_{1z}^{HMD[1]} = -\frac{IA}{4\pi}\cos\phi \begin{cases} \int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho \rho)\left(e^{ik_{1z}|z|} - \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right) & z \geq 0 \\ \int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho \rho)\left(-e^{ik_{1z}|z|} - \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right) & z < 0 \end{cases}$$

$$E_{1z}^{HMD[1]} = \frac{\omega\mu_1 IA}{4\pi}\sin\phi \int_0^\infty dk_\rho \frac{k_\rho^2}{k_{1z}} J_1(k_\rho \rho)\left(e^{ik_{1z}|z|} + \tilde{R}_{12}^{TM} e^{ik_{1z}(z+2d_1)}\right)$$

$$H_{1\rho}^{HMD[1]} = -i\frac{IA}{4\pi}\cos\phi \int_0^\infty dk_\rho k_{1z} k_\rho \left[J_0(k_\rho \rho) - \frac{J_1(k_\rho \rho)}{k_\rho \rho}\right]\left(e^{ik_{1z}|z|} - \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right)$$

$$-ik_1^2 \frac{IA}{4\pi\rho}\cos\phi \int_0^\infty dk_\rho \frac{1}{k_{1z}} J_1(k_\rho \rho)\left(e^{ik_{1z}|z|} + \tilde{R}_{12}^{TM} e^{ik_{1z}(z+2d_1)}\right)$$

$$H_{1\phi}^{HMD[1]} = i\frac{IA}{4\pi\rho}\sin\phi \int_0^\infty dk_\rho k_{1z} J_1(k_\rho \rho)\left(e^{ik_{1z}|z|} - \tilde{R}_{12}^{TE} e^{ik_{1z}(z+2d_1)}\right)$$

$$+ik_1^2 \frac{IA}{4\pi}\sin\phi \int_0^\infty dk_\rho \frac{1}{k_{1z}} k_\rho \left[J_0(k_\rho \rho) - \frac{J_1(k_\rho \rho)}{k_\rho \rho}\right]\left(e^{ik_{1z}|z|} + \tilde{R}_{12}^{TM} e^{ik_{1z}(z+2d_1)}\right)$$

$$E_{1\rho}^{HMD[1]} = -i\frac{\omega\mu_1 IA}{4\pi}\sin\phi \int_0^\infty dk_\rho k_\rho \left[\frac{J_1(k_\rho \rho)}{k_\rho \rho}\left(\tilde{R}_{12}^{TM} + \tilde{R}_{12}^{TE}\right) - J_0(k_\rho \rho)\tilde{R}_{12}^{TM}\right] e^{ik_{1z}(z+2d_1)}$$

$$-i\frac{\omega\mu_1 IA}{4\pi}\sin\phi \begin{cases} -\int_0^\infty dk_\rho k_\rho J_0(k_\rho \rho) e^{ik_{1z}|z|} & z \geq 0 \\ \int_0^\infty dk_\rho k_\rho J_0(k_\rho \rho) e^{ik_{1z}|z|} & -d_1 \leq z < 0 \end{cases}$$

$$E_{1\phi}^{HMD[1]} = -i\frac{\omega\mu_1 IA}{4\pi}\cos\phi \int_0^\infty dk_\rho k_\rho \left[J_0(k_\rho \rho)\tilde{R}_{12}^{TE} - \frac{J_1(k_\rho \rho)}{k_\rho \rho}\left(\tilde{R}_{12}^{TE} + \tilde{R}_{12}^{TM}\right)\right] e^{ik_{1z}(z+2d_1)}$$

$$-i\frac{\omega\mu_1 IA}{4\pi}\cos\phi \begin{cases} -\int_0^\infty dk_\rho k_\rho J_0(k_\rho \rho) e^{ik_{1z}|z|} & z \geq 0 \\ \int_0^\infty dk_\rho k_\rho J_0(k_\rho \rho) e^{ik_{1z}|z|} & -d_1 \leq z < 0 \end{cases}$$

Fields in Region 2: ($-d_1 > z \geq -d_2$)

$$H_{2z}^{HMD[1]} = \frac{\mu_1 IA}{4\pi\mu_2}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1\left(k_\rho\rho\right)A_2^{TE}\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$E_{2z}^{HMD[1]} = \frac{\omega\mu_1 IA}{4\pi}\frac{\varepsilon_1}{\varepsilon_2}\sin\phi\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{1z}} J_1\left(k_\rho\rho\right)A_2^{TM}\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$H_{2\rho}^{HMD[1]} = i\frac{\mu_1 IA}{4\pi\mu_2}\cos\phi\int_0^\infty dk_\rho A_2^{TE}k_{2z}k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(-e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$-ik_2^2\frac{\mu_1 IA}{4\pi\rho\mu_2}\frac{\varepsilon_1}{\varepsilon_2}\cos\phi\int_0^\infty dk_\rho A_2^{TM}\frac{1}{k_{1z}}J_1\left(k_\rho\rho\right)\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$H_{2\phi}^{HMD[1]} = i\frac{\mu_1 IA}{4\pi\rho\mu_2}\sin\phi\int_0^\infty dk_\rho A_2^{TE}k_{2z}J_1\left(k_\rho\rho\right)\left(e^{-ik_{2z}z} - \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$+ik_2^2\frac{\mu_1 IA}{4\pi\mu_2}\frac{\varepsilon_1}{\varepsilon_2}\sin\phi\int_0^\infty dk_\rho\frac{A_2^{TM}k_\rho}{k_{1z}}\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$E_{2\rho}^{HMD[1]} = -i\frac{\omega\mu_1 IA}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho A_2^{TE}J_1\left(k_\rho\rho\right)\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$+i\frac{\omega\mu_1 IA}{4\pi}\frac{\varepsilon_1}{\varepsilon_2}\sin\phi\int_0^\infty dk_\rho\frac{A_2^{TM}k_{2z}k_\rho}{k_{1z}}\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(-e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

$$E_{2\phi}^{HMD[1]} = -i\frac{\omega\mu_1 IA}{4\pi}\cos\phi\int_0^\infty dk_\rho A_2^{TE}k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{-ik_{2z}z} + \tilde{R}_{23}^{TE}e^{ik_{2z}(z+2d_2)}\right)$$

$$+i\frac{\omega\mu_1 IA}{4\pi\rho}\frac{\varepsilon_1}{\varepsilon_2}\cos\phi\int_0^\infty dk_\rho\frac{A_2^{TM}k_{2z}}{k_{1z}}J_1\left(k_\rho\rho\right)\left(-e^{-ik_{2z}z} + \tilde{R}_{23}^{TM}e^{ik_{2z}(z+2d_2)}\right)$$

HED in Region 2

Located at $z = -z_0$ - Interface 1-2 at $z = -d_1$ - Interface 2-3 at $z = -d_2$

Fields in Region 1: ($z \geq -d_1$)

$$E_{1z}^{HED[2]} = i\frac{Il}{4\pi\omega\varepsilon_1}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1\left(k_\rho\rho\right)e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM}B_2^{TM}$$

$$H_{1z}^{HED[2]} = i\frac{Il}{4\pi}\frac{\mu_2}{\mu_1}\sin\phi\int_0^\infty dk_\rho\frac{k_\rho^2}{k_{2z}}J_1\left(k_\rho\rho\right)e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE}B_2^{TE}$$

$$E_{1\rho}^{HED[2]} = -\frac{Il}{4\pi\omega\varepsilon_1}\cos\phi\int_0^\infty dk_\rho k_{1z}k_\rho\left[J_0\left(k_\rho\rho\right)-\frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM}B_2^{TM}$$

$$-k_1^2\frac{Il}{4\pi\omega\varepsilon_1\rho}\frac{\mu_2}{\mu_1}\cos\phi\int_0^\infty dk_\rho\frac{1}{k_{2z}}J_1\left(k_\rho\rho\right)e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE}B_2^{TE}$$

$$E_{1\phi}^{HED[2]} = \frac{Il}{4\pi\omega\varepsilon_1\rho}\sin\phi\int_0^\infty dk_\rho k_{1z}J_1\left(k_\rho\rho\right)e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM}B_2^{TM}$$

$$+k_1^2\frac{Il}{4\pi\omega\varepsilon_1}\frac{\mu_2}{\mu_1}\sin\phi\int_0^\infty dk_\rho\frac{1}{k_{2z}}k_\rho\left[J_0\left(k_\rho\rho\right)-\frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE}B_2^{TE}$$

$$H_{1\rho}^{HED[2]} = -\frac{Il}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho J_1\left(k_\rho\rho\right)e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM}B_2^{TM}$$

$$-\frac{Il}{4\pi}\frac{\mu_2}{\mu_1}\sin\phi\int_0^\infty dk_\rho\frac{k_{1z}}{k_{2z}}k_\rho\left[J_0\left(k_\rho\rho\right)-\frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE}B_2^{TE}$$

$$H_{1\phi}^{HED[2]} = -\frac{Il}{4\pi}\cos\phi\int_0^\infty dk_\rho k_\rho\left[J_0\left(k_\rho\rho\right)-\frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM}B_2^{TM}$$

$$-\frac{Il}{4\pi\rho}\frac{\mu_2}{\mu_1}\cos\phi\int_0^\infty dk_\rho\frac{k_{1z}}{k_{2z}}J_1\left(k_\rho\rho\right)e^{ik_{1z}(z+d_1)}e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE}B_2^{TE}$$

Fields in Region 2 <u>above</u> the source: ($-d_1 > z \geq -z_0$)

$$E_{2z}^{HED[2]} = i\frac{Il}{4\pi\omega\varepsilon_2}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1\left(k_\rho\rho\right)B_2^{TM}\left(e^{ik_{2z}z}+e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$H_{2z}^{HED[2]} = i\frac{Il}{4\pi}\sin\phi\int_0^\infty dk_\rho\frac{k_\rho^2}{k_{2z}}J_1\left(k_\rho\rho\right)B_2^{TE}\left(e^{ik_{2z}z}+e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

$$E_{2\rho}^{HED[2]} = -\frac{Il}{4\pi\omega\varepsilon_2}\cos\phi\int_0^\infty dk_\rho k_{2z}B_2^{TM}k_\rho\left[J_0\left(k_\rho\rho\right)-\frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{ik_{2z}z}-e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$-k_2^2\frac{Il}{4\pi\omega\varepsilon_2\rho}\cos\phi\int_0^\infty dk_\rho\frac{B_2^{TE}}{k_{2z}}J_1\left(k_\rho\rho\right)\left(e^{ik_{2z}z}+e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

$$E_{2\phi}^{HED[2]} = \frac{Il}{4\pi\omega\varepsilon_2\rho}\sin\phi\int_0^\infty dk_\rho k_{2z} B_2^{TM} J_1(k_\rho\rho)\left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$+k_2^2\frac{Il}{4\pi\omega\varepsilon_2}\sin\phi\int_0^\infty dk_\rho \frac{k_\rho B_2^{TE}}{k_{2z}}\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

$$H_{2\rho}^{HED[2]} = -\frac{Il}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho B_2^{TM} J_1(k_\rho\rho)\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$-\frac{Il}{4\pi}\sin\phi\int_0^\infty dk_\rho B_2^{TE} k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

$$H_{2\phi}^{HED[2]} = -\frac{Il}{4\pi}\cos\phi\int_0^\infty dk_\rho B_2^{TM} k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$-\frac{Il}{4\pi\rho}\cos\phi\int_0^\infty dk_\rho B_2^{TE} J_1(k_\rho\rho)\left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

Fields in Region 2 <u>below</u> the source: ($-z_0 \geq z > -d_2$)

$$E_{2z}^{HED[2]} = i\frac{Il}{4\pi\omega\varepsilon_2}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho\rho) C_2^{TM}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$H_{2z}^{HED[2]} = i\frac{Il}{4\pi}\sin\phi\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{2z}} J_1(k_\rho\rho) C_2^{TE}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$E_{2\rho}^{HED[2]} = -\frac{Il}{4\pi\omega\varepsilon_2}\cos\phi\int_0^\infty dk_\rho k_{2z} k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right] C_2^{TM}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$-k_2^2\frac{Il}{4\pi\omega\varepsilon_2\rho}\cos\phi\int_0^\infty dk_\rho \frac{1}{k_{2z}} J_1(k_\rho\rho) C_2^{TE}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$E_{2\phi}^{HED[2]} = \frac{Il}{4\pi\omega\varepsilon_2\rho}\sin\phi\int_0^\infty dk_\rho k_{2z} J_1(k_\rho\rho) C_2^{TM}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$+k_2^2\frac{Il}{4\pi\omega\varepsilon_2}\sin\phi\int_0^\infty dk_\rho \frac{k_\rho}{k_{2z}}\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right] C_2^{TE}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$H_{2\rho}^{HED[2]} = -\frac{Il}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho J_1\left(k_\rho\rho\right)C_2^{TM}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$-\frac{Il}{4\pi}\sin\phi\int_0^\infty dk_\rho k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]C_2^{TE}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$H_{2\phi}^{HED[2]} = -\frac{Il}{4\pi}\cos\phi\int_0^\infty dk_\rho k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]C_2^{TM}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$-\frac{Il}{4\pi\rho}\cos\phi\int_0^\infty dk_\rho J_1\left(k_\rho\rho\right)C_2^{TE}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

HMD in Region 2

Located at $z = -z_0$ - Interface 1-2 at $z = -d_1$ - Interface 2-3 at $z = -d_2$

Fields in Region 1: ($z \geq -d_1$)

$$E_{1z}^{HMD[2]} = \frac{\omega\mu_2 IA}{4\pi}\frac{\varepsilon_2}{\varepsilon_1}\sin\phi\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{2z}} J_1(k_\rho\rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

$$H_{1z}^{HMD[2]} = -\frac{\mu_2 IA}{4\pi\mu_1}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho\rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE} M_2^{TE}$$

$$E_{1\rho}^{HMD[2]} = \frac{i\omega\mu_2 IA}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho J_1(k_\rho\rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE} M_2^{TE}$$

$$+\frac{i\omega\mu_2 IA}{4\pi}\frac{\varepsilon_2}{\varepsilon_1}\sin\phi\int_0^\infty dk_\rho \frac{k_{1z}}{k_{2z}} k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right] e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

$$E_{1\phi}^{HMD[2]} = \frac{i\omega\mu_2 IA}{4\pi}\cos\phi\int_0^\infty dk_\rho k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right] e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE} M_2^{TE}$$

$$+\frac{i\omega\mu_2 IA}{4\pi\rho}\frac{\varepsilon_2}{\varepsilon_1}\cos\phi\int_0^\infty dk_\rho \frac{k_{1z}}{k_{2z}} J_1(k_\rho\rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

$$H_{1\rho}^{HMD[2]} = \frac{i\mu_2 IA}{4\pi\mu_1}\cos\phi\int_0^\infty dk_\rho k_{1z} k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right] e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE} M_2^{TE}$$

$$+k_1^2\frac{i\mu_2 IA}{4\pi\mu_1\rho}\frac{\varepsilon_2}{\varepsilon_1}\cos\phi\int_0^\infty dk_\rho \frac{1}{k_{2z}} J_1(k_\rho\rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

$$H_{1\phi}^{HMD[2]} = \frac{i\mu_2 IA}{4\pi\mu_1\rho}\sin\phi\int_0^\infty dk_\rho k_{1z} J_1(k_\rho\rho) e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TE} M_2^{TE}$$

$$+k_1^2\frac{i\mu_2 IA}{4\pi\mu_1}\frac{\varepsilon_2}{\varepsilon_1}\sin\phi\int_0^\infty dk_\rho \frac{1}{k_{2z}} k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right] e^{ik_{1z}(z+d_1)} e^{-ik_{2z}d_1}\tilde{T}_{21}^{TM} M_2^{TM}$$

Fields in Region 2 <u>above</u> the source: ($-d_1 > z \geq -z_0$)

$$E_{2z}^{HMD[2]} = \frac{\omega\mu_2 IA}{4\pi}\sin\phi\int_0^\infty dk_\rho \frac{k_\rho^2}{k_{2z}} J_1(k_\rho\rho) M_2^{TM}\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)} R_{21}^{TM}\right)$$

$$H_{2z}^{HMD[2]} = -\frac{IA}{4\pi}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1(k_\rho\rho) M_2^{TE}\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)} R_{21}^{TE}\right)$$

$$E_{2\rho}^{HMD[2]} = \frac{i\omega\mu_2 IA}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho M_2^{TE} J_1(k_\rho\rho)\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)} R_{21}^{TE}\right)$$

$$+\frac{i\omega\mu_2 IA}{4\pi}\sin\phi\int_0^\infty dk_\rho M_2^{TM} k_\rho\left[J_0(k_\rho\rho) - \frac{J_1(k_\rho\rho)}{k_\rho\rho}\right]\left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)} R_{21}^{TM}\right)$$

$$E_{2\phi}^{HMD[2]} = \frac{i\omega\mu_2 IA}{4\pi}\cos\phi\int_0^\infty dk_\rho M_2^{TE} k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

$$+\frac{i\omega\mu_2 IA}{4\pi\rho}\cos\phi\int_0^\infty dk_\rho M_2^{TM} J_1\left(k_\rho\rho\right)\left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$H_{2\rho}^{HMD[2]} = -\frac{iIA}{4\pi}\cos\phi\int_0^\infty dk_\rho k_{2z} M_2^{TE} k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

$$-k_2^2\frac{iIA}{4\pi\rho}\cos\phi\int_0^\infty dk_\rho\frac{M_2^{TM}}{k_{2z}} J_1\left(k_\rho\rho\right)\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

$$H_{2\phi}^{HMD[2]} = \frac{iIA}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho k_{2z} M_2^{TE} J_1\left(k_\rho\rho\right)\left(e^{ik_{2z}z} - e^{-ik_{2z}(z+2d_1)}R_{21}^{TE}\right)$$

$$+k_2^2\frac{iIA}{4\pi}\sin\phi\int_0^\infty dk_\rho\frac{k_\rho M_2^{TM}}{k_{2z}}\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right]\left(e^{ik_{2z}z} + e^{-ik_{2z}(z+2d_1)}R_{21}^{TM}\right)$$

Fields in Region 2 <u>below</u> the source: ($-z_0 \geq z > -d_2$)

$$E_{2z}^{HMD[2]} = \frac{\omega\mu_2 IA}{4\pi}\sin\phi\int_0^\infty dk_\rho\frac{k_\rho^2}{k_{2z}} J_1\left(k_\rho\rho\right) N_2^{TM}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$H_{2z}^{HMD[2]} = -\frac{IA}{4\pi}\cos\phi\int_0^\infty dk_\rho k_\rho^2 J_1\left(k_\rho\rho\right) N_2^{TE}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$E_{2\rho}^{HMD[2]} = \frac{i\omega\mu_2 IA}{4\pi\rho}\sin\phi\int_0^\infty dk_\rho J_1\left(k_\rho\rho\right) N_2^{TE}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$+\frac{i\omega\mu_2 IA}{4\pi}\sin\phi\int_0^\infty dk_\rho k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right] N_2^{TM}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$E_{2\phi}^{HMD[2]} = \frac{i\omega\mu_2 IA}{4\pi}\cos\phi\int_0^\infty dk_\rho k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right] N_2^{TE}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$+\frac{i\omega\mu_2 IA}{4\pi\rho}\cos\phi\int_0^\infty dk_\rho J_1\left(k_\rho\rho\right) N_2^{TM}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$H_{2\rho}^{HMD[2]} = -\frac{iIA}{4\pi}\cos\phi\int_0^\infty dk_\rho k_{2z} k_\rho\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right] N_2^{TE}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$-k_2^2\frac{iIA}{4\pi\rho}\cos\phi\int_0^\infty dk_\rho\frac{1}{k_{2z}} J_1\left(k_\rho\rho\right) N_2^{TM}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

$$H_{2\phi}^{HMD[2]} = \frac{iIA}{4\pi\rho}\sin\phi\int_0^{\infty} dk_\rho k_{2z} J_1\left(k_\rho\rho\right) N_2^{TE}\left(-e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TE}\right)$$

$$+k_2^2\frac{iIA}{4\pi}\sin\phi\int_0^{\infty} dk_\rho \frac{k_\rho}{k_{2z}}\left[J_0\left(k_\rho\rho\right) - \frac{J_1\left(k_\rho\rho\right)}{k_\rho\rho}\right] N_2^{TM}\left(e^{-ik_{2z}z} + e^{ik_{2z}(z+2d_2)}R_{23}^{TM}\right)$$

Appendix B - Contents of CD-ROM

On the CD-ROM, named **ELF_VLF** delivered with this report are three folders/subdirectories:

Hill_King, Menu_Interface, Sommerfeld_Routines

In addition to these folders/subdirectories is a PDF file version of this Report

ELF_Report.PDF"

Below are listed the files contained in each of the folders/subdirectories

Hill_King:

HillFig10.m
HillFig6.m
HillFig7.m
HillFig8.m
HillFig9.m
KingsGreenFtn.m

Menu_Interface:

BSDefinition.m
BSDefinitionBox.m
BSDefinitionBox.mat
BSDEllipsoid.m
BSDEllipsoidBox.m
BSDEllipsoidBox.mat
BSDWireStruct.m
BSDWireStructBox.m
BSDWireStructBox.mat
ellipplot.m
SRLGlobals.m
SRLGrid.m
SRLGridBox.m
SRLGridBox.mat
SRLObject.m
SRLObjectBox.m
SRLObjectBox.mat
SRLocations.m
SRLocationsBox.m
SRLocationsBox.mat
SRLPrivates.m

Sommerfeld_Routines:

EHEDRegion1_1.m
EHEDRegion1_2.m

EHEDRegion2_1.m
EHEDRegion2_2.m
EHMDRegion1_1.m
EHMDRegion1_2.m
EHMDRegion2_1.m
EHMDRegion2_2.m
EVEDRegion1_1.m
EVEDRegion1_2.m
EVEDRegion2_1.m
EVEDRegion2_2.m
EVMDRegion1_1.m
EVMDRegion1_2.m
EVMDRegion2_1.m
EVMDRegion2_2.m
ExampleHED.m
fastInt.m
fastIntSk1z.m
getElecDipoleField.m
getkrInterp.m
getMagDipMomCond.m
getMagDipoleField.m
HHEDRegion1_1.m
HHEDRegion1_2.m
HHEDRegion2_1.m
HHEDRegion2_2.m
HHMDRegion1_1.m
HHMDRegion1_2.m
HHMDRegion2_1.m
HHMDRegion2_2.m
HVEDRegion1_1.m
HVEDRegion1_2.m
HVEDRegion2_1.m
HVEDRegion2_2.m
HVMDRegion1_1.m
HVMDRegion1_2.m
HVMDRegion2_1.m
HVMDRegion2_2.m
interp1COS.m
LoopSensorExample.m
psiN.m
SommerfeldGlobals.m
SommerfeldInit.m